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This longitudinal study is one of a series which is devoted to the task of evaluating new, secondary school mathematics curricula. This series of studies has involved ninth and eleventh grade teachers teaching a new mathematics curriculum (School Mathematics Study Group, University of Illinois Committee on School Mathematics, or Ball State Program) in schools in five state (Minnesota, North and South Dakota, Iowa, and Wisconsin). While teaching the new curricula, some of the teachers were provided with an in-service training course in foundations of modern algebra. This report concerns attempts (1) to separate the effects on student performance attributable to the new curricula from effects attributable to the teachers, and (2) to relate differences in teacher effectiveness to differences in subject matter knowledge as measured by performance in the in-service training course. Among the findings reported are (1) teacher effects tended to dominate curriculum effects, and (2) relations were found between gains of classes and general measures of teachers' subject-matter competence. (RP)

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FINAL REPORT
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PROGRAMMED CORRESPONDENCE COURSES IN ALGEBRA AND GEOMETRY FOR INSERVICE TEACHER TRAINING: FIELD STUDIES

August, 1968

U.S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

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Wells Hively II

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Minnesota National Laboratory
Minnesota State Department of Education

Saint Paul, Minnesota

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INTRODUCTION

This study is one of a series, carried out by the Minnesota National Laboratory since 1961, devoted to evaluating the major, new, secondary-school mathematics curricula. This series of studies has involved teachers in Minnesota, North Dakota, South Dakota, Iowa, and Wisconsin, and has therefore come to be called the "Five-State Project". Other reports in the series include Ericksen & Ryan, 1966; Kraft, 1963; Ryan, 1966, 1968a, 1968b; Ryan & Rising, 1966a, 1966b; Wright, 1967.

The practical problem to which the present study was directed is as follows. How do the characteristics of mathematics teachers currently in service affect the introduction of new mathematics curricula? Specifically, what is the role of the teachers' knowledge of the foundations of the modern subject matter? A sizeable sample of ninth and eleventh-grade mathematics teachers in the Upper Midwest was followed as they began to teach a new curriculum (SMSG, Illinois, or Ball State), and their classes were tested for several consecutive years. At the same time, some of the teachers were furnished with an inservice-training course in foundations of modern algebra. This report concerns attempts to separate the effects on student performance attributable to the new curricula from effects attributable to the teachers, and to relate differences in teacher effectiveness to differences in subject-matter knowledge as measured by performance on the inservice-training course. The longitudinal nature of the study made it possible, in addition, to estimate the effects of experience with a new curriculum on the teachers' effectiveness with the same materials in later years, and also (since some teachers continued to teach a "conventional" class as well) to estimate the effects of experience with a new curriculum on the teachers' effectiveness with conventional materials. An attempt was made to estimate the effects of the program of inservice-training itself upon the effectiveness of the teachers, but that turned out to be impossible within the limits of the procedures used.

A companion study was planned, utilizing an inservice-training course in foundations of modern geometry, but, for reasons outlined in Chapter Two, sufficient data could not be gathered on teachers' performances in the geometry course to provide interpretable correlations with the performances of students.

The research program was based on a rather traditional concept of "curriculum materials" and of the role of a teacher in a self-contained classroom. The curriculum materials are thought of as syllabi which suggest, but do not specify completely, the activities of the teacher and the students. The teacher is expected to "follow" the text but to modify and supplement as he sees fit. This arrangement represents a midpoint on the range between a purely programmed, or self-instruction, curriculum (where there would be few teacher effects) and a situation in which each teacher concocts his own course materials (where teacher effects would be maximized). The current trend in curriculum research and development is away from this general, teacher-syllabus situation toward systems of instruction in which more specific roles of the teacher are defined in relation to the roles of specific materials and devices. In the latter context, research on teacher effectiveness tends to focus upon specialized teacher roles rather than on the search for the "ideal teacher"(c.f. Getzels and Jackson, 1963, p.533). However, the teacher-syllabus situation typifies current educational practice in secondary mathematics and is likely to continue to do so for some time to come. Therefore, it is important to assess whatever global effects may be operating in this situation, to provide school administrators information to guide them in introducing new mathematics programs and in selecting and training teachers to teach them, as well as to guide the allocation of resources for further curriculum development and teacher training. If, for example, one found little difference among curricula in their effects on student performance but large and stable differences among teachers, one might be tempted to step up activities in selection and training of teachers. If, on the other hand, the effects of curricula were large and teacher differences small, or if teacher effects diminished quickly with experience, one might be tempted to distribute promising curricula widely without worrying too much about related problems of teacher preparation. If neither curricula nor teachers had pronounced and consistent effects (a not uncommon finding in educational research) one might reason that a more precise specification of the activities composing the curricula is needed, along the lines of a "*systems*" *approach*.

For typical discussions of teacher and curriculum effects in relation to new mathematics and science programs see the Review of Educational Research, Vol 34, No. 3, June 1964; also the conference report edited by Rosenbloom (1964). For an excellent review of the literature and a description of the line of thinking leading up to the present series of studies see Torrance et al. (1966).

Related Research

The following reviews and critical analyses of the literature on characteristics of teachers were found especially useful: Ackerman (1954), Barr (1948), Biddle and Ellena (1964), Castetter, Standlee, and Fattu (1954), Domas and Tiedeman (1950), Gage (1963), Gage and Unruh (1967), Getzels and Jackson (1963), LeFevre (1967), Mitzel and Gross (1956), Rabinowitz and Travers (1953), Remmers et al. (1953), and Watters (1954).

The present study falls into the "criterion-of-effectiveness paradigm" as distinguished from the "teaching-process paradigm" by Gage (1963). That is, knowledge of the foundations of modern algebra is conceived to be a relatively stable, measurable characteristic of teachers which manifests itself in a variety of actions appropriate to a variety of situations. No attempt is made to observe what the teachers actually do with their students: measures of the hypothetical characteristic are simply correlated with measures of student performance. Previous studies of this type have not been notably informative. There are at least three possible reasons why: (1) the generality of the effects studied; (2) the lack of longitudinal studies; and (3) inadequacy of the measures used.

In summarizing his 1963 review, Gage concluded:

> "...we may make better progress if we develop 'micro-criteria' of effectiveness. At the very least, such an approach would imply that effectiveness be sharply specified in terms of subject matter and grade level."

Despite this recommendation, our review of the literature since 1960 turned up only nine studies of teacher effectiveness which were specifically directed to mathematics. Of these, studies utilizing direct measures of teachers' subject-matter competence were found only at the elementary-school level.

Weak and conflicting findings from the secondary-school surveys of Leonhardt (1963) and Sparks (1960) tend to support Rosenbloom's (1961) claim that information taken from mathematics-teachers' records (training institutions, course credits, years of experience, etc.) is of little or no use in predicting relative effectiveness. Studies by Clark (1968) and Lampela (1966), in which teachers were given tests of mathematics achievement, showed no correlations with students' mathematics achievement in grades 2, 4, 5, 6, and 7. However, Bassham (1962) found significant positive correlations between measures of teachers' knowledge of mathematics and students' gains in sixth grade. Dickens (1966) and Hand (1967) found that inservice-training in modern mathematics for elementary school teachers had no effect upon pupil achievement. However, Houston and DeVault (1963) found positive correlations between teachers' gains in an inservice program and gains of their students. There is little in these studies, however, which may be generalized to the new, high-school, mathematics programs.

Despite the enormous literature on teacher characteristics, and the implicit assumption that these characteristics are relatively stable over time, we found only one study in which mathematics teachers were followed for more than one year. Kraft (1963) found that gains on the STEP test did not provide stable indices of relative teacher effectiveness in grades 10, 11, and 12 across successive years (a finding which was confirmed in the present study).

In the entire literature, we succeeded in finding only one other longitudinal study of relative teacher effectiveness (Morsh, Burgess, & Smith, 1956). Before putting much faith in correlations of teacher characteristics with measures of teachers' effectiveness, it seems reasonable to ask how stable are the measured differences in teachers' effectiveness under particular field conditions. The closely related problem of developing adequate measures of student (and teacher) subject-matter knowledge is discussed at some length in later sections of this report.

Chapter 1

A STUDY OF THE RELATIVE EFFECTIVENESS OF HIGH SCHOOL ALGEBRA TEACHERS IN RELATION TO MEASURES OF THEIR PROFICIENCY IN FOUNDATIONS OF MODERN ALGEBRA.

The general purpose of this study was to gather descriptive, longitudinal data from ninth and eleventh grade classes participating in the Five-State Project and to look for relationships between performance of the classes and the subject-matter competence of the teachers. The following specific questions were examined:

1. Do some teachers ordinarily have better incoming students than others? That is, is there some stability from year to year in the entering ability of mathematics classes taught by particular teachers in particular schools in comparison with other teachers in other schools? One would expect to find relatively stable differences in incoming performance due to socio-economic and geographic selective factors, but the magnitude of these differences is not known.

2. Is there any apparent trend in the entering ability of students in successive years?

3. Do more mathematically able teachers tend to have more mathematically able students? This might happen, for example, if socio-economic and geographic selective factors operated similarly on both students and teachers.

4. Do some teachers consistently teach better than others? That is do the gains of students in classes taught by particular teachers tend to be stable from year to year in relation to the gains of students in classes taught by other teachers? If stable differences are found are they related to the curricula used by the teachers?

5. Is there any trend in the gains over successive years? An upward trend might be expected, for example, if the teachers profited from the experience of teaching a modern curriculum.

6. Is there any relationship between the teachers' subject matter competence, as measured by performance on the in-service-training course, and their effectiveness as measured by the gains of their classes?

Note that these questions are interrelated. The overall pattern is more important than the statistical significance of any particular set of comparisons.

Method

Sample of Teachers

The population of ninth and eleventh-grade teachers participating in the Five-State Project was composed by asking for volunteers from among teachers in Minnesota, North Dakota, South Dakota, Iowa and Wisconsin to teach one of four possible modern-mathematics secondary-school curricula: the SMSG, Illinois, Maryland, and Ball State Programs. The request for volunteers was sent through the cooperating State Departments of Education of the neighboring states. A condition upon entry was that the teachers should not have taught a "modern" mathematics curriculum before. This population was formed in 1961. During the academic year 1961-62 all teachers continued to teach the "conventional" curriculum then in use at their school while their classes were tested. During the academic year 1962-63, the teachers were asked to teach a class using one of the experimental texts while continuing to teach a class from their conventional text (schools were asked to assign students to these two classes at random). Many teachers, however, were unable to do this and instead taught only an experimental class. In 1963-64 some of the teachers moved up with their classes to the next grade, teaching either an experimental class or both an experimental and conventional class as before. Other teachers remained at the same grade level and taught either experimental or experimental and conventional classes to other groups of incoming students. They continued to follow this pattern in 1964-65. Approximately 20% of the teachers discontinued work in the project each year and these were replaced by additional recruitment as follows. In 1962, a "second phase" of teachers entered the project following the pattern established by the teachers in Phase One. A third phase was added in 1963 and a fourth in 1964. Table 1 summarizes the participation and Figure 1 shows the geographic distribution of the participants over the period 1961-62 through 1965-66. Appendix A lists the year of participation and curricula taught by each teacher, together with the name and location of his school.

It is important to remember that teachers volunteered to participate in the project and chose the experimental curricula they would like to teach. This was basically a naturalistic, rather than an experimental study.

Measurement of Teacher's Subject-Matter Competence

Content of the inservice-training course. The course of inservice-training in foundations of modern algebra (produced under a separate grant from the National Science Foundation) was programmed for self-instruction and sent to the teachers via the Correspondence Study Department of the University of Minnesota. The following is a description of its content by the senior author, Ancel C. Mewborn, University of North Carolina.

> The course is designed to raise the level of competence in algebra of secondary school teachers of mathematics. The content was not chosen with any particular "modern" curriculum or text in mind; however the main factor in the choice of topics

was the relevance of these topics to material usually taught in high school algebra. The course is keyed more closely to the content of high school algebra, i.e., to the algebra of the real number system, than are most courses designed for teacher training. There is little emphasis on abstract algebraic systems --- groups, rings, fields, etc. --- for their own merit, although examples of these are presented to clarify the algebraic properties of the real number system.

Considerable emphasis is placed on the logical structure of algebra. The real number system is developed from a set of axioms, and most of the important properties of the system which are studied in high school algebra are derived as theorems from the axioms. Much emphasis is placed on the relationships existing between various real number properties and on concepts which tend to unify, or tie together, these properties. Very little emphasis is placed on the development of manipulative skills, since it is assumed that those taking the course will already possess these skills. Thus, for example, it is assumed that the student is able to perform the steps in solving linear or higher degree polynomial equations, but that he may not fully understand the logical reasoning involved in the process. The emphasis is on the logical basis for the process. One notable exception to the above is in the solving of inequalities. Here the student is given considerable practice in the manipulative skills involved.

A great deal of effort is devoted to teaching the nature of proof in algebra. The student is expected to learn to construct for himself proofs of many of the easier theorems. He is also expected to be able to follow the steps in proofs of more difficult theorems when they are presented to him. The ability to construct proofs is not a primary goal of the course. Thus it is not so important that the student be able to prove that if $a \neq 0$, the equation $ax = b$ has a unique solution as that he understand how this theorem is related to the theorem that the equation $a + x = b$ has a unique solution. Having the student construct proofs of these two theorems is regarded as an effective technique for teaching him this relation.

No separate treatment is given to logic or logical reasoning in connection with the construction of proofs. The few essential ideas of logic are worked into the basic material. For example, instead of a detailed treatment of "proof by contradiction", examples of this kind of proof are given and the essential details of the nature of such a proof are pointed out in connection with the examples. Within reasonable bounds, emphasis is placed on preciseness in formulating definitions and in stating theorems.

The course begins with a brief discussion of sets, relations, and functions. The algebra of sets -- unions, intersection, complementation, etc. -- is not considered essential for the development of high school algebra and is not included in the course. A relation is defined as a set of ordered repairs, and a function is defined as a special kind of relation. These notions are illustrated by numerous examples taken from high school algebra and, in some instances, high school geometry. Many of these examples are used to illustrate how functions and relations may be defined by equations and inequalities. The method of defining a function by specifying its domain and a rule of correspondence is emphasized.

A rather detailed development of the real number system is given, beginning with the field axioms for the real numbers. Examples of algebraic systems different from the real number system but sharing many of its properties are introduced to help clarify these properties. Order axioms for the real number system are introduced, and an extended discussion of inequalities and absolute values, based on these axioms, is included. Finally, the completeness of the system of real numbers is given by the least upper bound axiom. This is used to give a precise meaning to infinite decimal expansions.

Following the development of the real numbers separate treatments are given to the natural numbers, integers, and rational numbers. Each of these is defined axiomatically as a subsystem of the real number system. Included is an extensive discussion of mathematical induction.

The complex numbers are defined in terms of real numbers. In this, as in earlier sections, there is a strong geometric orientation.

After a brief discussion of the algebra of real functions on a set there is an extensive treatment of polynomials. Here there is great emphasis on the unique factorization properties which are introduced earlier in connection with the system of integers. The parallel existing between the domain of integers and the domain of real polynomials is emphasized.

> In the part of the course farthest removed from the usual high school curriculum some elementary properties of equivalence relations and groups are derived. This is the only place in the course where a type of algebraic system is treated, to any appreciable degree, abstractly (i.e., outside the context of some particular example).
>
> Finally, it should be emphasized that the course is designed for the teacher of algebra to help clarify for him the foundations of high school algebra. The presentation is not considered appropriate for the high school student, nor is the course appropriate as a model for the design of a high school algebra course. It is hoped that the increased understanding gained from this course will help the teacher to utilize more effectively one of the existing texts designed for a high school course.

Two versions of the course were administered: the first during the academic year 1963-64 and the second, revised version during the academic year 1964-65. (A report of the development and evaluation of the course is available from the Minnesota Academy of Science, 3100 38th Avenue South, Minneapolis, Minnesota 55406.) Table 2 presents a breakdown of the course content in the two versions. Although the content was not substantially changed in the revised version it was divided into shorter units of instruction, each followed by a shorter test, but supplemented by occasional review tests over larger sections.

Administration of the inservice-training course. Early in the fall of 1963 a letter was sent to all teachers in Phases One and Two (see Table 1) asking for volunteers to work on the course. This letter included a short description of the content and indicated that it would be equivalent to a three quarter-hour, upper-level, college mathematics course. Of the 147 ninth and eleventh-grade teachers then participating in Phases One and Two, 76 volunteered. For reasons unrelated to the present study these were randomly divided into two groups stratified according to grade, phase, and experimental curriculum. The 1963 edition of the algebra program was sent to one of these groups and a letter was sent to members of the other group thanking them for volunteering but telling them that they would receive a revised edition of the program in 1964. The selected teachers were then sent the first unit of material. After completing the unit and taking the unit test they returned both the unit and test to the Correspondence Study Department. They were then sent the next unit together with a fresh copy of the first unit to use for review. The test for the first unit was sent to a grader and returned with comments. This usually took from one to two weeks. This procedure continued until the teacher had finished all seven units. He was then sent an extensive final examination to take at home and a shorter final examination to take under the supervision of his school principal.

In the fall of 1964 the revised edition was sent to all of the teachers then participating in the Five-State Project who had not already worked on the 1963 edition. Volunteers were not solicited: the first unit of the course was simply sent, together with a letter requesting the teacher to work on the course as much as possible. The method of administration was the same as in 1963 except that the first three units were sent at the same time. When the teacher returned Unit 1 he was sent Unit 4 together with a fresh copy of Unit 1, and so on. This kept them from having to wait for new materials.

All tests were graded by a member of the team of writers who developed the course, with the assistance of graduate students in Mathematics at the University of Minnesota.

Specific measures. Several measures were extracted from the performance of teachers on the correspondence course. Because of differences in course format and administration procedures, these measures are parallel but not exactly the same for the two versions.

1. Volunteering (1963-64). Whether or not a teacher volunteered to take the course presumably reflects his need for subject-matter help. (This is a nominal variable consisting of two categories.)

2. Returns of the first unit (1964-65). This applies only to teachers in Phases Three and Four who did not have the previous opportunity to volunteer. It is comparable to volunteering except that the teachers were probably able to form a better judgment of the content and difficulty of the course from an examination of the first three units. If they did not elect to work on the course, it could have been either because they did not feel the need or because the material appeared too difficult. (Also a nominal variable of two categories.)

3. Score on the test for the first unit (1963). This score reflects both mathematical ability and prior training. To the extent that it correlates with performance on tests over subsequent units in the course, it may be thought of as reflecting the teacher's ability to learn the new material. The average correlation of the test for the first unit with the tests for the following five units was .62. (An ordinal scale of 38 points.)

4. Total score through Test A (1964). This measure is comparable to the one preceding except that, because the unit tests were shorter in the revised version, it was necessary to sum across the first section in order to obtain scores which were relatively reliable as predictors of subsequent performance. The average intercorrelation of total scores for the four sections of the 1964 version was .54. (An ordinal scale of 83 points.)

5. Working time on the first unit (1963-64). Each teacher maintained a log of the time it took to work through each unit of the course. In general, working times were uncorrelated with test performance but rather well intercorrelated in themselves, indicating stable individual differences among the teachers. Working time may thus serve as another possible index of subject matter competence: two teachers may learn essentially the same amount from the course but one may learn faster than the other. The average intercorrelation of working time on the first unit with working time on the other five units of the course was .40. (A ratio scale with units of 30 minutes.)

6. Working time on the first unit (1964-65). This measure is identical to the one preceding. The average intercorrelation of working times on the first unit with working times on the other 13 units of the course in 1964 was .70. Note that revision of the course increased the reliability of the working times as measures of individual differences while decreasing the reliability of the test scores. In 1964 there was a slight tendency for long working times to be associated with higher scores. This is reflected in an average correlation for the 14 units of .23. (A ratio scale with units of 5 minutes.)

Measures of Student Performance

The classes of each teacher were routinely tested at the beginning of the fall and late in the spring with machine-scoreable, group tests. The test used in the fall was the mathematics section of the Sequential Tests of Educational Progress (STEP). Both the ninth and eleventh grades received Form 2A. These constituted the pretest measures, reflecting a large component of general mathematics ability. Form 2B of the STEP test was given in the spring as a posttest during each year from 1961 through 1964. This was supplemented in 1964 by a special set of tests constructed by the Minnesota National Laboratory to sample a range of specific modern mathematics content at both the ninth and eleventh grade levels. In 1965 revised forms of the Minnesota National Laboratory tests were given and the STEP posttests were replaced by the Cooperative Mathematics Tests (COOP). The COOP tests given in the ninth grade were Structure of the Number System (Form A) and Algebra I (Form B). Those used in the eleventh grade were Algebra II (Form A) and Algebra III (Form A).

Specific measures.

1. Median and range of pretest raw scores for each class. These are treated as points on an ordinal scale. (In general, to facilitate the interpretation of a large number of comparisons, for some of which it was unsafe to assume normal distributions of scores, the median has been used as the estimate of central tendency and nonparametric statistics have been applied.)

2. Median posttest raw scores of classes, corrected for regression on the STEP pretest. This was done by calculating the regression of each posttest on the pretest, utilizing the scores from all students in a given grade who had taken a given pair of pre- and posttests, in all years in which that pair of tests was given. Table 3 lists each of the posttests for which a regression equation was obtained, the years over which the data were pooled, the numbers of students involved, and the equations obtained. These equations were used to predict the median posttest score to be expected from each class, given the median pretest score, under the assumption that the conditions under which they worked were uniform. The deviation of the observed, median posttest score from the score predicted from the regression equation was chosen as the best general measure of relative variation in teaching conditions. A class taught by a particularly effective teacher (or an effective curriculum) would be expected to yield a large positive deviation score. These scores are also treated as points on an ordinal scale.

Results

Pretest Data

Distributions and relative stability. Figures 2 through 9 show the distributions of pretest scores for classes taught by ninth and eleventh-grade teachers during the period 1961-62 through 1964-65. By using Appendix A one can examine the relationship between entering ability and geographic location.

Table 4 shows the relative stability of pretest scores for classes taught by the same teachers in consecutive years. The groups were formed by taking the largest number of ninth-grade conventional classes taught by the same teacher in two consecutive years (9C-2) and for three consecutive years (9C-3); then doing the same thing for the ninth-grade experimental classes (9E-2, 9E-3), and so on for the eleventh grade. With the exception of the eleventh-grade conventional classes there was considerable stability in the rank ordering of the classes by pretest scores across consecutive years. Some teachers did ordinarily have better incoming students than others.

Trends. Table 5 shows the group trends in pretest scores in terms of the median of the class medians obtained in each year. In both ninth and eleventh-grade groups there was little variation and no apparent trend across years.

Relation of teacher characteristics to pretest scores. In Table 6 the median pretest scores of classes taught during the academic year 1963-64, by teachers who volunteered for the inservice-training course in algebra, are compared with those of classes taught by teachers who did not volunteer. In Table 7 the median pretest scores of classes taught during the academic year 1964-65, by teachers who returned at least the first unit of the course, are compared with those of classes taught by teachers who did not return any units. There were no significant differences or systematic patterns in either year: teachers' interest in the inservice-training course does not appear to have been related to the entering ability of their classes.

Table 8 shows correlations between teachers' scores on the first units of the inservice-training course and the median pretest scores of their classes, for those teachers who worked on the course in either 1963-64 or 1964-65. Table 9 shows similar correlations using the teachers' working times on the first unit. None of the correlations in either table are significant. In general, there were no systematic relationships between these measures of teachers' subject-matter competence and the entering ability of their students. Within the range of ability represented by the teachers who chose to work on the inservice-training course, there is no evidence that more mathematically able teachers tended to have more mathematically able students.

Posttest Data Obtained from the STEP Tests

Relative stability. Table 10 shows the relative stability of posttest deviation scores for classes taught by the same teachers in consecutive years, utilizing the same groups as in Table 4. A comparison of these tables shows that the posttest deviation scores (which are supposed to reflect systematic effects of instruction) were much less stable than the pretest scores. As measured by the STEP test, few teachers consistently taught better than others. This was the case despite the fact that, in this analysis, differences among teachers were confounded with differences in curricula (within experimental and conventional groups).

Trends. Table 11 shows the group trends in posttest deviation scores on the STEP test, utilizing the same groups presented in Table 5. Since the curricula used by the teachers remained the same during these successive years, changes in the deviation scores may be interpreted as rough indices of changes in the teachers' effectiveness. (Although some teachers may not have taught consistently better than others, all of them might have become more effective through experience with the new curricula.) The trend for ninth-grade conventional classes was downward, but that for the experimental classes was upward. No systematic trends were apparent in the eleventh grade.

Relations of teacher characteristics to deviation scores. In Table 12 the median posttest deviation scores of classes taught during the academic year 1963-64, by teachers who volunteered for the inservice-training course, are compared with those classes taught by teachers who did not volunteer. In Table 13 the median posttest deviation scores of classes taught during the academic year 1964-65, by teachers who returned at least the first unit are compared with those of classes taught by teachers who did not return any units. In group 9C, teachers who did not volunteer for the inservice-training course had classes who achieved significantly more than classes of teachers who volunteered. This finding (which is plausible in that the volunteers might be accurately expressing their need) was not supported by the overall pattern of group differences. In general, teachers' interest in the inservice-training course was not very systematically related to their effectiveness in teaching either conventional or experimental curricula, as measured by the STEP test.

Table 14 shows correlations between teachers' scores on the first units of the inservice-training course and the posttest deviation scores of their classes. The correlations were symmetrically distributed around zero. Within the range of teachers who chose to work on the inservice-training course there was no relation between this measure of subject-matter competence and effectiveness as measured by the STEP tests.

Table 15 shows correlations between teachers' working times on the first unit of the inservice-training course and the posttest deviation scores of their classes. These correlations were low, but systematically negative, indicating a mild relationship in which more rapid progress through the inservice-training course was associated with greater effectiveness.

Posttest Data Obtained from the Minnesota National Laboratory Tests

Relative stability. Tests especially constructed by the Minnesota National Laboratory (MNL) to sample the content of the modern mathematics programs were administered as posttests in the academic year 1964-65 and revised versions of the same tests were administered again as posttests in 1965-66. Table 16 shows the relative stability of the deviation scores obtained from these tests for classes taught by the same teachers in these two consecutive years. Comparison with Table 10 shows that the results obtained from the MNL tests were more stable than those obtained from the STEP test. As measured by the MNL tests, some teachers did teach consistently better than others.

In the foregoing table, teacher effects are confounded with the effects of different curricula. Table 17 shows the relative stability of the posttest deviation scores obtained from the MNL tests, separately for each of the experimental curricula. The magnitudes of the correlation coefficients obtained within curricula were approximately the same as those obtained by pooling across curricula, except in the Illinois program. The relative stability of the scores appears to have been primarily due to effects associated with the teachers rather than to effects associated with the different curricula.

Another way of looking at the pervasiveness of the teacher effects, in relation to effects of different curricula, is to correlate the posttest deviation scores for experimental and conventional classes taught by the same teachers. These data are shown in Table 18. In the ninth grade, it appears that some teachers taught better than others no matter what curriculum they taught. The eleventh-grade data are difficult to interpret because of the small number of classes involved.

Relationship of teacher characteristics to deviation scores. In Table 19 the median, posttest deviation scores obtained from the MNL tests from classes taught during the academic year 1964-65 by teachers who returned at least the first unit of the inservice-training course are compared with those of classes taught by teachers who did not return any units. The results are similar to those obtained from the STEP tests (Table 13) in that no interpretable pattern is apparent. Table 20 shows correlations between teachers' scores on the first units of the inservice-training course and the median, posttest deviation scores of their classes obtained from the MNL tests in the same academic year. A comparison with Table 14 shows that the results from the MNL tests yielded more consistent positive correlations, although none were significant. Table 21 shows correlations between teachers' working times on the first unit of the course and the median posttest deviation scores of their classes obtained from the MNL tests in the same academic year. Comparison with Table 15 shows a similar trend toward low negative correlations.

Relationship of teacher characteristics to posttest performance of classes in 1965-66. The correlations presented above were carried out on measures obtained from teachers and students during the same academic years. If the characteristics of teachers, measured by their performance on the inservice-training course were relatively stable, they should yield similar correlational patterns with student achievement in subsequent years. Table 22 shows correlations of teachers' scores on the first units of the inservice-training course (in 1964-65) with the median posttest deviation scores of their classes in 1965-66. The correlations show no interpretable pattern: the mild positive trend previously obtained (Table 20) did not hold up across the following year.

Table 23 shows similar correlations for teachers' working times on the first unit of the course. The pattern of negative correlations is again apparent, and the magnitudes are somewhat greater. Teachers who worked through the inservice-training course rapidly appeared to have been consistently somewhat more effective than those who worked slowly.

Interactions between teacher characteristics and student ability. It is plausible to suppose that a teacher who knows a good deal about algebra might pass more of his knowledge on to upper-ability students than to students of lower ability. Correlations between measures of teachers' knowledge and students' gains might therefore be higher for upper-ability students than for the class as a whole. Bassham (1962) found such an effect in elementary-school mathematics. To explore this possibility, classes were subdivided into upper and lower ability groups by splitting each class at its median on the pretest. Some separate correlations were then run for these subgroups. The results for the MNL tests are shown in Tables 24 and 25.

Table 24 shows correlations between teachers' scores and students' gains. In 9C and 11E, the correlations for the subgroups mirrored the correlations for classes as shown in Table 20. In 9E and 11C, the correlations were higher for the lower-ability subgroups.

Table 25 shows correlations between teachers' working times and student gains (compare Table 21). In all grades and curricula negative correlations were larger for the lower-ability subgroups than for those of upper-ability. Overall, the previously discussed correlations between the measures of teacher subject-matter competence and student gains were somewhat greater for lower than for upper-ability students.

Discussion

The results of this study place a great deal of emphasis upon the achievement tests constructed by the Minnesota National Laboratory. Trends obtained from the STEP test tended to agree with those obtained from the MNL tests, but were less pronounced and stable. An examination of the items from the MNL tests, copies of which appear in Appendix B, shows them to be much more directly content-relevant than the items in the STEP tests. The MNL tests represent a sample of specific knowledge and skill rather than a test of general problem solving ability. Thus, the results of the foregoing studies indicate that some teachers rather consistently communicate more, relatively specific, subject matter than others. The fact that these differences among teachers held up across experimental and conventional curricula suggests that many teachers were carrying over a considerable amount of "modern" content into their conventional courses. The fact that the differences were equally pronounced within the experimental curricula themselves suggests, however, that it is not so much a matter of carry-over as of domination of curriculum effects by teacher effects.

The items in the MNL tests had not been selected according to statistical criteria (e.g. item-test correlations). Therefore, they represent a more heterogeneous variety of skills and abilities than commonly found in standardized tests of mathematics achievement. This heterogeneity may make them more sensitive to specific teaching conditions. In general, one can argue that the procedures of statistical item selection tend to create homogeneous tests, with heavy loadings on basic mathematical ability, and that such tests are necessarily insensitive to differences among specific teaching procedures or curricula. Betts (1935) made this point some time ago, but the problem of content sampling vs. normative scaling remains a basic unresolved issue in the construction of tests for curriculum evaluation (see, for example, Review of Educational Research, Vol. 34, No. 3, June, 1964, p. 302).

It is important to note that the MNL tests were generally difficult. The means and variances were relatively low. It is possible that the obtained results represent relatively trivial differences in specific content which certain teachers typically covered and other teachers did not. Preliminary analysis of the items reveals no obvious artifacts of this sort, but further detailed analysis is needed.

The present report is not the place for such an analysis. It is hoped that a technical report on these tests will soon be published by the Minnesota National Laboratory.

Unfortunately, no longitudinal data were collected with the COOP tests. It would be interesting to compare the results from them with the results from the MNL tests. In general, it is unfortunate that the bulk of the longitudinal data were collected with the STEP tests. Revisions in the MNL tests made it impossible to follow trends across the two years in which the tests were given, and therefore the important question of whether a teacher's effectiveness may change through exposure to a modern mathematics curriculum could not be examined with anything but the insensitive STEP test.

Conclusions

The data from the eleventh grade showed less consistent patterns than did the data from the ninth grade, partly because of differences in the numbers of classes involved. Therefore the following conclusions apply most directly to ninth-grade algebra classes.

Stable differences in the effectiveness of teachers, measured in terms of a rather direct content sampling of general curriculum goals, were found. These manifested themselves both in correlations across successive years and in correlations between modern and conventional classes taught by the same teachers. The magnitudes of these correlations were relatively substantial, when one takes into consideration the general noise-level of field studies of this type.

Mild but consistent relations were found between measures of relative teacher effectiveness and general measures of teachers' subject-matter competence, i.e., the relative speed and thoroughness with which teachers themselves learned new mathematics content. These relations between relative teacher effectiveness and subject-matter competence were somewhat more clearly reflected in the performance of lower-ability students than in those of upper-ability (an unexpected finding). It was impossible to determine, from these studies, what other characteristics of the teachers, besides subject-matter competence, contributed to the differences in their effectiveness.

The results of these studies say very little about the effects of specific training or experience upon relative teacher effectiveness. Results using the STEP test did not show that experience with a modern curriculum improved teachers' effectiveness, either in the modern curriculum itself or in a conventional curriculum. However, the pattern of other evidence suggests that the STEP test is insensitive to nearly all independent variables. Longitudinal, experimental studies incorporating more effective testing procedures are needed to further investigate the effects of specific teacher training or experience. In general, the results of these studies suggest that (in the typical teacher-syllabus situation) emphasis upon selection and training of teachers is at least as important as the development of new syllabi.

Chapter 2

RESPONSES OF HIGH SCHOOL GEOMETRY TEACHERS TO A PROGRAMMED CORRESPONDENCE COURSE IN FOUNDATIONS OF MODERN GEOMETRY.

A study of the relative effectiveness of tenth-grade geometry teachers, in relation to their performance on a programmed correspondence course in foundations of modern geometry, was planned as a companion to the foregoing study of ninth and eleventh-grade algebra teachers. However, the programmed course in geometry turned out to be too difficult for the sample of teachers to whom it was sent. Too few teachers succeeded in completing enough units to provide usable measures of relative subject-matter competence.

The following is a description of the course written by the senior author, Murray S. Klamkin, Ford Scientific Laboratory. The course (developed under a grant from the National Science Foundation) is still undergoing revision. (Information about it may be obtained from the Minnesota Academy of Science, 3100 38th Ave. South, Minneapolis 6, Minn.)

> The course is not designed with any particular text in mind. Topics were chosen mainly for their relevance to material taught in high school geometry. However, a primary aim of the course is to treat geometry across a broad frontier, indicating its connections with other branches of mathematics. Therefore, much material has been included which is not usually mentioned in a high school geometry course, e.g., vector geometry; area, volume, and measure; continuity; convexity; symmetry; and inequalities and maxima and minima.
>
> A variety of physical motivating material has been included, wherever possible, in order to enable students to obtain an intuitive "feel" for the basic concepts. Preciseness in formulating definitions and in stating theorems is stressed, but there has been a conscious attempt to prevent "rigor mortis" from setting in. For example, when proving theorems in the unit on plane separation the student is expected to make explicit all the steps involving plane separation axioms even if they may be intuitively obvious, but "simple" steps involving material from previous units, may be sketched in.
>
> The final unit of the course includes a large amount of material on "Problem Solving a la Polyá". These problem solving materials, which are intended to be used concurrently with the other units of the course, provide an extensive

introduction to problem solving strategies together with a wide variety of "open-ended" problems.

A brief description of the contents is as follows:

Unit I - Introduction
    List of Problems
Unit II - Incidence
Unit III - Distance
    Betweeness
    Collinear Sets
    Plane Separation
    Congruence
Unit IV - Angular Measure
    Congruent Triangles
Unit V - Perpendicular Lines and Planes
    Geometric Inequalities
Unit VI - The Parallel Postulate and Non-Euclidean Geometries
    Lobachevskian Geometry
    Riemannian Geometry
    Geometry and Physical Space
    Euclidean Geometry
    Parallel Lines and Planes in Three Dimensions
    Dihedral Angles and Perpendicular Planes
Unit VII - Polygons
    Interior and Exterior
    Polygonal Regions
    Polyhedral Angles
    Polyhedra
Unit VIII - Similarity in 2 and 3 Dimensions
    Homotheticity
Unit IX - Circles and Spheres
    Spherical Geometry
    Geometrical Constructions
Unit X - Area
    Dimensionality and Units
    Equidecomposition and Equicomplementation
    Limits
Unit XI - Volume
Unit XII - Problem Solving
    A. Patterns:
        (a) Psychological Blocks
        (b) Symmetry
        (c) Relaxation
        (d) Induction
        (e) Inequalities and Maxima and Minima
        (f) Continuity
        (g) Convexity

B. Philosophy and Applications of Transform Theory:
   - (a) Orthogonal Projection
   - (b) Conical Projection and Principle of Duality
   - (c) Inversion
   - (d) Equivalence Classes
   - (e) Miscellaneous Transforms

In the fall of 1965 the course was sent to approximately sixty tenth-grade teachers participating in the Five-State Project and also to a sample of 199 tenth-grade teachers from the Five-State Project area, who were not then participating in the project. The latter sample was chosen as follows. Announcements were sent (through the cooperating State Departments of Education) to all High School superintendents in the five states. 675 teachers applied. Only teachers who were going to be teaching tenth-grade geometry during the 1965-66 academic year were considered eligible. There were 465 of these, from which a random sample proportional to the number of high school teachers in each state was randomly drawn: 64 teachers from Minnesota, 49 from Iowa, 11 from North Dakota, 11 from South Dakota, and 64 from Wisconsin.

The course was administered through the Correspondence Study Department of the University of Minnesota following a procedure similar to that described for the Programmed Course in Algebra. Table 26 shows the numbers of teachers returning each unit from November, 1965 to April, 1967.

Chapter 3

CONTENT ANALYSIS OF THE PROGRAMMED COURSE IN ALGEBRA FOR TEACHERS IN RELATION TO MODERN, HIGH-SCHOOL CURRICULUM MATERIAL

The research reported in Chapter 1 dealt with very general measures of teacher subject-matter competence and student performance in algebra. A detailed analysis of these relationships, in terms of specific concepts and processes, is badly needed to supplement study of the general relationships. As part of the present project, it was proposed that such an analysis be initiated by carrying out a detailed study of the connections between the contents of the Minnesota National Laboratory's Algebra Course for Teachers and the contents of one or more of the modern, high-school curricula. It was orginally proposed that this be done with the SMSG, ninth and eleventh-grade algebra courses, but it was decided that a more practical approach would be to perform the analysis on widely used commercial textbooks which paralleled the SMSG courses rather closely. The textbooks chosen were Modern Algebra: Structure and Method - Book I by Dolciani, Berman, and Freilich; and Modern Algebra: Structure and Method - Book Two by Dolciani, Berman, and Wooton (both published by Houghton-Mifflin).

Appendix C contains the results of this content analysis in the form of tables of contingencies between detailed topic headings in the texts. This analysis was carried out by a member of the writing team for the Minnesota National Laboratory course. The contingencies have been scaled as follows. Where the topics covered were identical or equivalent, the cell in the table representing the intersection was marked with the letter A. Where the topics were closely related, the cell was marked with the letter B. (Usually this meant that the treatment of the topic was more general in the course for the teachers than in the course for the children.) Where the topics were distantly related, the cell was marked with the letter C.

Clusters of entries show blocks of mutally related content. By following columns in the tables, one can see how particular material in the course for the teachers is related to the range of material in the courses for the children, and by following rows, one can do the reverse. Summing across columns and rows is a convenient way of summarizing the general relations. Such summaries are presented in Appendix C, pages C-12 - C-18. In general, the pattern of contingencies shows that most of the material in the MNL course is related to topics from Dolciani et al., Book 2. The major exception is Unit 14, on Equivalence Relations and Groups. Other topics, for which specific contingencies were not recorded, are systematically related to topics where substantial contingencies were noted.

For example, although "The Algebra of Real Functions" and "Polynomial Functions" were not specifically related to material in Book 2, they are systematically connected to other material on polynomials which was.

Although nearly all of the material in the MNL course is related to material in Book 2, not all of the material in Book 2 is related to that in the MNL course. Unrelated topics include exponential functions and logarithms, trigonometric functions, arithmetic and geometric progressions, and probability.

The contingencies between topics are fewer, and less direct, for Dolciani et al., Book 1. They are scattered, however, so that few topics in the course for the teachers and the course for the children are completely unrelated.

How might a content analysis of this type help in investigating specific relationships between teacher subject-matter competence and student performance? Its main function is to aid in constructing tests and in making predictions. Where a contingency between topics is categorized as identical or equivalent (class A), the performance of teachers and students can be measured with the same test items and correlated in a straightforward way: if Teacher X knows the topic and Teacher Y does not, are the students in Teacher X's class more likely to learn it than the students in the class taught by Teacher Y?

Where the topics are classified as related, but not equivalent (class B or class C) there may be two advantages for the teacher who possesses the more general knowledge. (1) He may be able to explain or answer questions more precisely than he otherwise would, and be less inclined to make inappropriate generalizations, mistakes or contradictions. This might result in better performance of his students on the restricted topics in their curriculum. This possibility may be tested in a straightforward way: if Teacher X knows more of a given topic than Teacher Y, do students in his class learn more of a given, more restricted topic than those in a class of Teacher Y? (2) Knowing the additional related material may make it possible for the teacher to "expand" the topic in the curriculum he is teaching, to provide enrichment to advanced students, and so forth. Testing this possibility is more complicated, because the teacher is, in effect, altering the curriculum for some students, and it is necessary to construct new test materials to cover these possible alterations.

RECOMMENDATIONS

The results of the present studies emphasize the importance, for systematic curriculum development and evaluation, of relatively large, longitudinal studies incorporating continuous development of new measures. An item for "research planning" was originally included in the proposal for this project to provide for a conference of mathematicians, researchers and educators devoted to the next generation of experimentation along these lines. Staff changes in the Minnesota National Laboratory make such a conference unfeasible at present. Consequently, the following notes are included here for the benefit of those who might wish to carry on further research and development along these lines.

It now seems most reasonable to begin with a single curriculum, such as is represented by the Dolciani et al. texts, and to deduce, from examination of this curriculum, the important related areas of "higher" mathematics. Following a content-sampling rationale for constructing achievement tests, the next step would be to create sizable test-item pools to cover topics in the curriculum, on one hand, and in the related areas of higher mathematics, on the other. (These should, of course, cover proof and problem-solving skills as well as routine content.) Items could then be drawn from these pools to create tests for the students and tests for the teachers, each stratified by topic. At the same time, tentative hypotheses of the following form ought to be developed: if Teacher A does better than Teacher B on Topic C, then students of A should do better than students of B on Topic C'.

In order to control for experience, it would probably be wisest to start with first-year teachers and to follow them longitudinally from that time on. It would be necessary to ask the teachers to use the chosen curriculum or to carry out experimentation in school systems which had adopted it. With close cooperation of the school system, it ought to be possible to ask the teachers to subject themselves routinely to an examination each year at the beginning of the Fall term. The students may then be tested at the end of the Spring term, and appropriate correlations computed.

It will be necessary, of course, to control for student ability. This may be done by testing the children at the beginning of the Fall term with a test of mathematical aptitude (and perhaps also with a test of prerequisites for the specific curriculum). If circumstances permit, these data might be used to stratify the students across teachers' classes. If the practical problems in this are insurmountable, then a procedure similar to the present regression analysis would have to be used. It does not seem impossible to set up the mechanics for carrying out routine longitudinal studies, following all the new teachers of mathematics in one or more school systems for a period of several years.

Special training programs may be introduced into this longitudinal context. For example, a selected sample of teachers might be contracted to spend the summer working on a course of instruction similar to that represented by the MNL Algebra Course. Effects of this training could then be assessed on the longitudinal baseline and in comparison with a control group of teachers who were not exposed to the same training program. (The most elegant study would compare the results from groups of teachers who were given different summer training programs, from which different effects upon students were predicted.)

Such a study should certainly incorporate other measures of teachers and students than those directly derived from content analysis of the curriculum, e.g., attitude and interest inventories, etc. (See Torrance, 1966).

To the extent that the chosen curriculum suggests specialized teachers' roles, this research should concentrate upon measures of effectiveness, and characteristics of teachers, specifically related to those roles. Longitudinal research of this type ought to involve continued, systematic alteration of both the curriculum and the measures in such a way as to provide developmental flexibility coupled with longitudinal continuity. This is a difficult, but not impossible, balance to achieve.

The practical outcome of a research and development program of this sort would be the development of inservice-training programs for teachers geared to specific curricula, texts, or systems of instruction. Under such a procedure teachers might very well "check-out" or become "certified" for work with specific curricula in more or less the same way that airline pilots are certified to fly particular types of aircraft. This research probably ought to be centered in school systems where the mathematics teachers and supervisors can become committed to it, rather than having it superimposed from outside. It results in a relatively subtle kind of teacher evaluation in terms of pupil performance.

SUMMARY

Ninth and eleventh-grade mathematics teachers in the Upper Midwest were followed as they began to teach a new mathematics curriculum (either SMSG, Illinois, or Ball State), and their classes were tested for several consecutive years. During this time some of the teachers were furnished with an inservice-training course in foundations of modern algebra. Attempts were made to separate the effects on student performance attributable to the new curricula from effects attributable to the teachers, and to relate differences in teacher effectiveness to differences in subject-matter knowledge as measured by performance on the inservice-training course.

Rank-orders of teachers, in terms of gains made by their classes, were significantly correlated across successive years. Some teachers consistently taught more than others did.

Teacher effects tended to dominate curriculum effects. When the same teachers taught both modern and conventional classes, their rank-orders in terms of gains by the modern classes were significantly correlated with their rank-orders in terms of gains by the conventional classes.

Mild but consistent relations were found between gains of classes and general measures of teachers' subject-matter competence, viz. the relative speed and thoroughness with which teachers themselves learned new mathematics content. Surprisingly, these relations were somewhat more clearly reflected in the performance of lower-ability students than in those of upper-ability.

The above findings were based mainly on data from a special set of achievement tests in ninth and eleventh-grade mathematics prepared by the Minnesota National Laboratory, tests which represent broad samples of modern-mathematics content, rather than scales of general mathematical ability. Results from the STEP test (a test of general ability) showed similar, but weaker trends.

A content analysis of the inservice-training course in algebra was carried out in relation to a popular, modern, ninth and eleventh-grade text series (Dolciani et al.), and a future line of research, based upon such detailed analyses, was proposed.

A companion study of tenth-grade geometry was not carried to completion because of difficulties with the inservice-training course.

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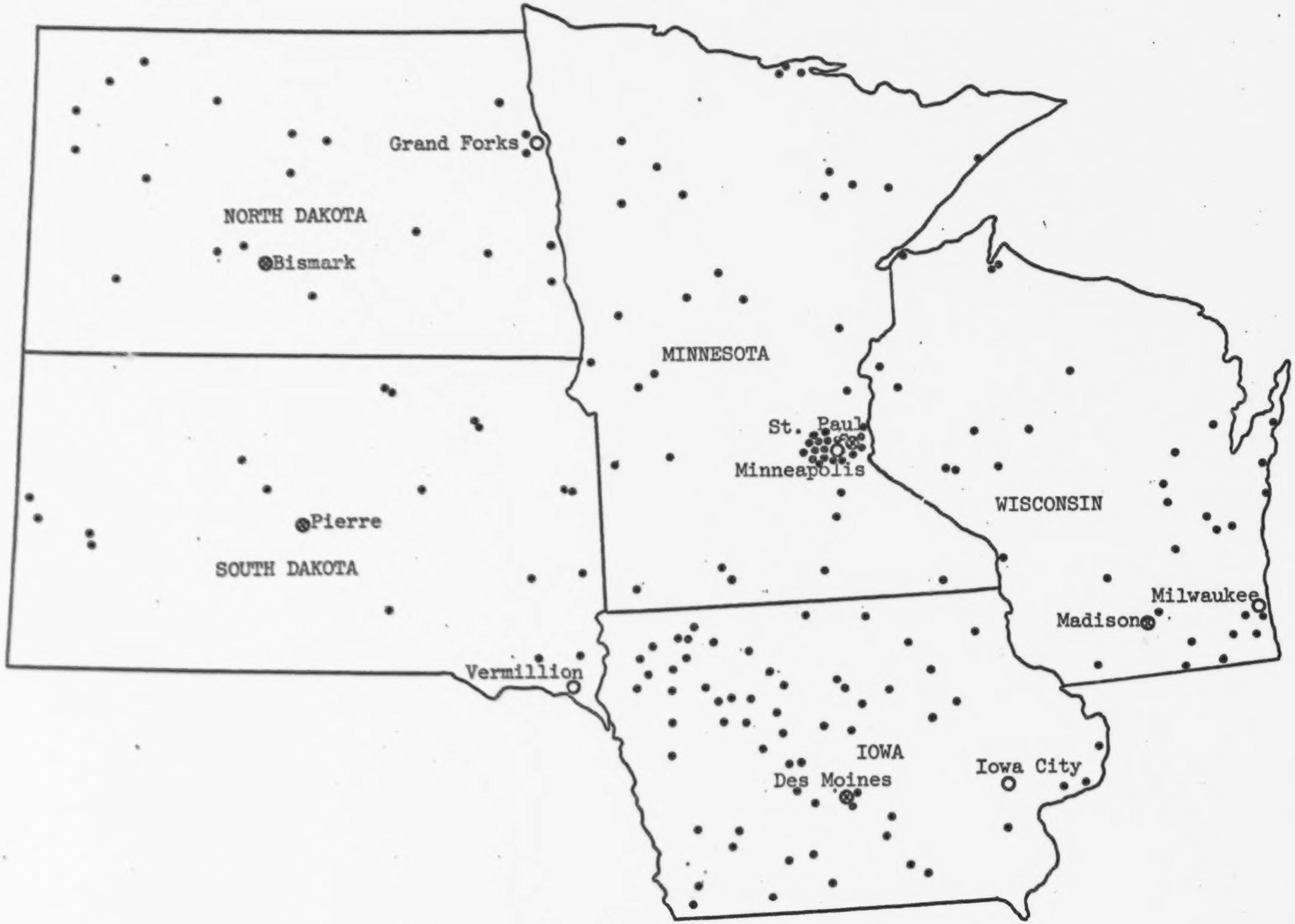

NORTH DAKOTA
Grand Forks
Bismark
SOUTH DAKOTA
Pierre
Vermillion
MINNESOTA
St. Paul
Minneapolis
WISCONSIN
Milwaukee
Madison
IOWA
Des Moines
Iowa City

Fig. 1. Geographic Distribution of Project Participants, 1961-62 through 1965-66.

											399											
											353											
											347											
											343											
											341											
											334											
											329				406							
									425		327			400	401							
								383	348		316			384	357							
							428	381	340	505	263		418	365	337	385						
							426	355	324	350	255	377	360	344	325	359		504				
			331	382			358	349	315	335	235	326	262	328	232	336		376		346	389	
	410	386	233	266			228	338	229	332	231	226	230	265	227	261		264		234	333	
12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34

Median Raw Score of Class on STEP Pretest (Form 2A)

Fig. 2. Frequency distribution of pretest median scores for 9th grade classes in 1961 (Conventional curricula only). Classes are identified by teacher I.D. numbers (See Appendix A).

Conventional Curricula

13	14	15	16	17	18	91	20	21	22	23	24	25	26	27	28	29	30
	619 240	692 617 331	228	751 722 707 591 338 328 310	743 689 569 344 265	694 676 505 426 340 334	629 425 377 358 264 261	739 721 713 701 656 651 608 332	729 717 688 675 666 648 632 400 327 325	761 718 687 683 680 639	730 711 708 664 655 644 618 615 360 227	737 734 357	758 740 685 677	703 667 384 326 255	662 359 278	347	

Median Raw Score of Class on STEP Pretest (Form 2A)

Experimental Curricula

13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
		331	338	337 310	386 265	400 397 358 334 300 278 229 228 226	425 340 336 335 324	428 406 360 347 304	426 416 390 357 348 334 332 240 235	505 439 344	377 365 328 234 227	431 385 343 325 261	504 384 359 326 255	350 327		389 264	

Median Raw Score of Class on STEP Pretest (Form 2A)

Fig. 3. Frequency distributions of pretest median scores for 9th grade classes in 1962. Classes are identified by teacher I.D. numbers (See Appendix A). I.D. numbers of teachers simultaneously teaching experimental and conventional classes are underlined.

Conventional Curricula

13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
		331	240	608 426	865 358	677 350	734 505 332 328 326 227	853 831 793 739 384	863 858 849 847 844 834 830 815 797 788 777 766 711 639 261	860 839 816 794 772 737 730 722	827 762 718 687 685 680 648 341	826 803 694	819 800 667	835 729 703 644			866 767				

Median Raw Score of Class on STEP Pretest (Form 2A)

Experimental Curricula

13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
	331			735 619	680 608 341 337	701 676 505 382	722 332 300 225	730 711 687 666 618 406 358 240	683 390 384 365 326 227	718 675 591 426 328 202	739 656 639 617 428	761 734 632 629 615 355	737 729 703 655 644 255	758 685 677 667 648 261 226	347 263	694 389 350		362 264		440	

Median Raw Score of Class on STEP Pretest (Form 2A)

Fig. 4. Frequency distributions of pretest median scores for 9th grade classes in 1963. Classes are identified by teacher I.D. numbers (See Appendix A). I.D. numbers of teachers simultaneously teaching experimental and conventional classes are underlined.

Conventional Curricula

13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
900	769	803		853 261	886 819 800 332	895 894 777 324	849 794 505	906 885 876 730 363	863 835 734 635 384 350	773 768 718 687	892 694 685 639 231 227		762 680 326	739		767 703					

Median Raw Score of Class on STEP Pretest (Form 2A)

Experimental Curricula

13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
900 803			839 735	865 769 762 332	850 849 739 730 676 324 226	858 853 830 591 337	930 777 632 609 224	797 772 384 229 202	920 863 813 800 793	906 896 761 694 505	819 768 687 686 639 600 326 263	917 834 831 815 773 734 693 635 628 438 363 231 227	703 282	835 818 261	680	718 710 685		929 794 350		767	264

Median Raw Score of Class on STEP Pretest (Form 2A)

Fig. 5. Frequency distributions of pretest median scores for 9th grade classes in 1964. Classes are identified by teacher I.D. numbers (See Appendix A). I.D. numbers of teachers simultaneously teaching experimental and conventional classes are underlined.

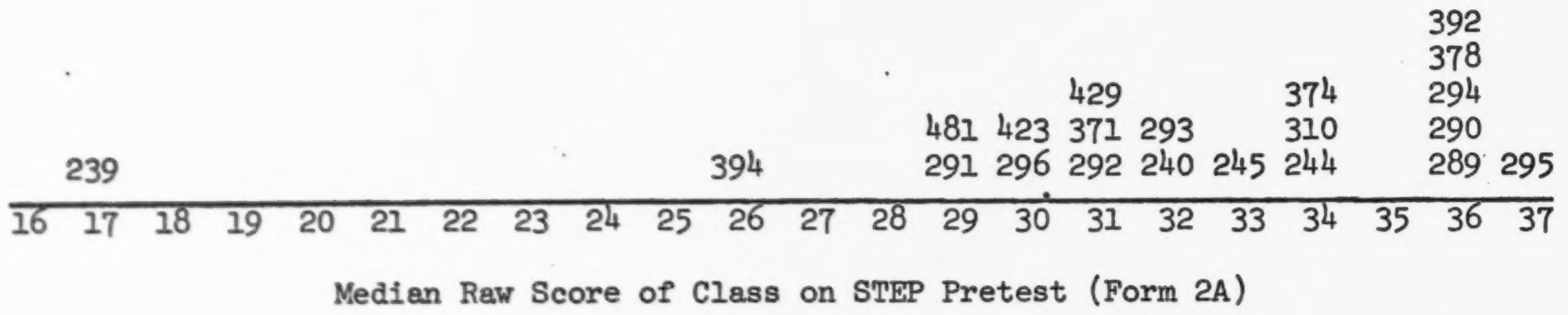
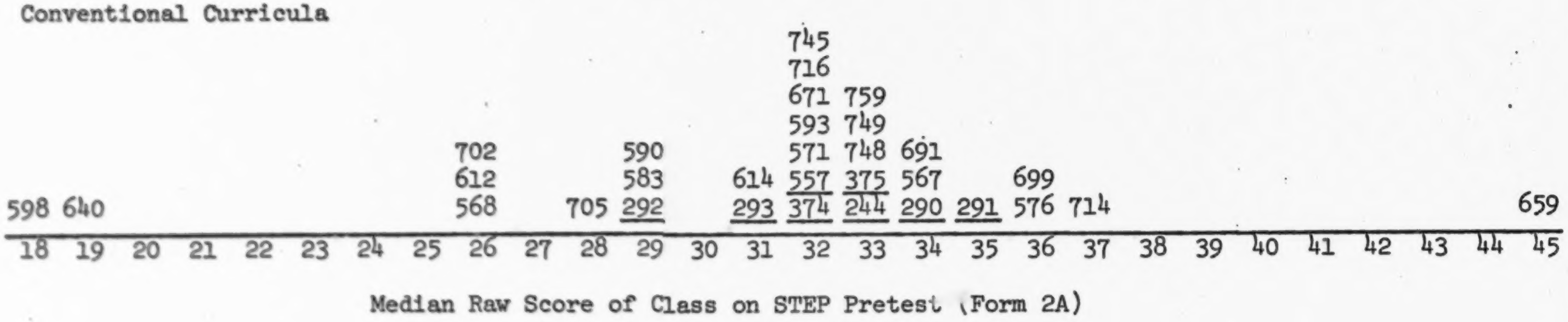


Fig. 6. Frequency distribution of pretest median scores for 11th grade classes in 1961. (Conventional curricula only). Classes are identified by teacher I.D. numbers (See Appendix A).


Conventional Curricula

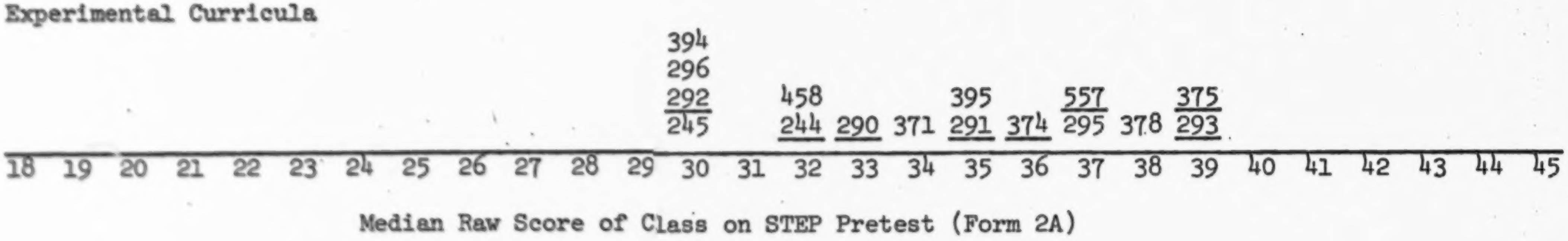

Experimental Curricula

Fig. 7. Frequency distributions of pretest median scores for 11th grade classes in 1962. Classes are identified by teacher I.D. numbers (See Appendix A). I.D. numbers of teachers simultaneously teaching experimental and conventional classes are underlined.

Conventional Curricula

Median Raw Score of Class on STEP Pretest (Form 2A)	Classes
18	769
19	
20	841
21	
22	
23	
24	
25	
26	845 778
27	
28	
29	805 306 292
30	859 854 791 748 419
31	825 824 745 612 375
32	855 840 764 749 716 691 671 236
33	818 781 759
34	832 702 593 583
35	821 699 557
36	
37	810
38	862 804
39	
40	802
41	
42	
43	659
44	

Median Raw Score of Class on STEP Pretest (Form 2A)

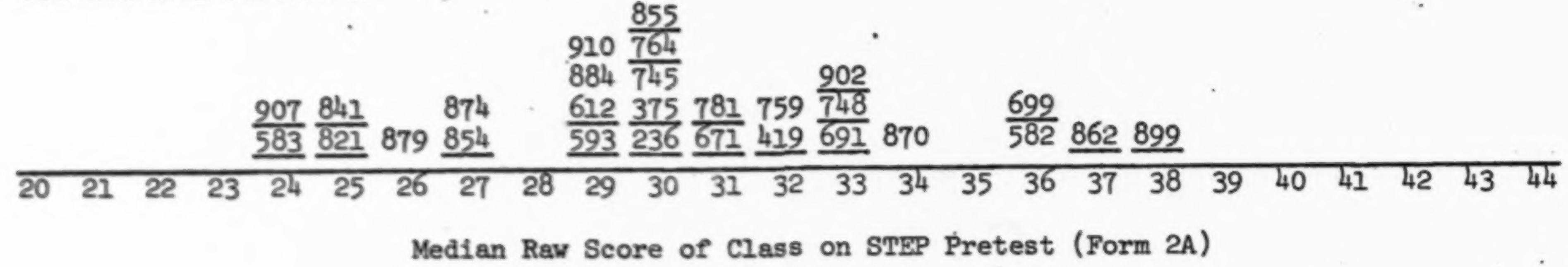
Experimental Curricula

Median Raw Score of Class on STEP Pretest (Form 2A)	Classes
18	
19	
20	
21	
22	238
23	
24	
25	
26	640 296
27	568
28	245
29	
30	590 306
3;	612
32	759 671 614 583 378 292
33	749 748 716 576 557 392
34	691 458 452
35	419 375 244
36	699
37	702 593 281 271 236
38	295
39	
40	745
41	659
42	
43	
44	

Median Raw Score of Class on STEP Pretest (Form 2A)

Fig. 8. Frequency distributions of pretest median scores for 11th grade classes in 1963. Classes are identified by teacher I.D. numbers (See Appendix A). I.D. numbers of teachers simultaneously teaching experimental and conventional classes are underlined.

Conventional Curricula



Experimental Curricula

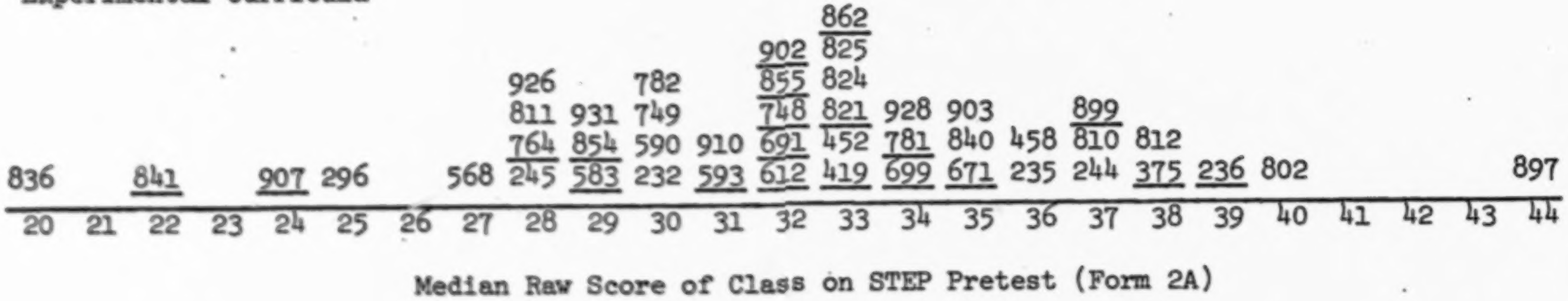


Fig. 9. Frequency distributions of pretest median scores for 11th grade classes in 1964. Classes are identified by teacher I.D. numbers (See Appendix A). I.D. numbers of teachers simultaneously teaching experimental and conventional classes are underlined.

Table 1

Number of 9th and 11th Grade Classes in Five-State Project by Year, Phase, Grade, and Curriculum

	1961-62		1962-63		1963-64		1964-65		1965-66	
Grade	9	11	9	11	9	11	9	11	9	11
Phase 1										
Conventional	120	38	34	11	14	6	10	3	5	2
Ball State	0	0	20	8	10	9	8	5	3	6
Illinois	0	0	18	0	9	0	4	0	3	0
SMSG	0	0	29	14	13	9	7	8	4	4
Classes:	N = 158		N = 134		N = 70		N = 45		N = 27	
Teachers:	N = 158		N = 89		N = 50		N = 32		N = 20	
Phase 2										
Conventional			56	26	19	12	10	12	7	2
Ball State			0	0	13	8	7	7	4	4
Illinois			0	0	10	0	5	0	4	0
SMSG			0	0	13	10	7	8	5	1
Classes:			N = 82		N = 85		N = 56		N = 27	
Teachers:			N = 82		N = 54		N = 34		N = 18	
Phase 3										
Conventional					35	22	14	9	6	1
Ball State					0	0	6	6	2	0
Illinois					0	0	6	0	3	0
SMSG					0	0	16	11	6	4
Classes:					N = 57		N = 68		N = 22	
Teachers:					N = 57		N = 45		N = 15	
Phase 4										
Conventional							12	10	5	3
Ball State							3	7	2	5
Illinois							3	0	4	0
SMSG							5	4	6	3
Classes:							N = 44		N = 28	
Teachers:							N = 34		N = 20	

Totals

	1961-62		1962-63		1963-64		1964-65		1965-66	
Grade	9	11	9	11	9	11	9	11	9	11
Conventional	120	38	90	37	68	40	46	34	23	8
Ball State	0	0	20	8	23	17	24	25	11	15
Illinois	0	0	18	0	19	0	18	0	14	0
SMSG	0	0	29	14	26	19	35	31	21	12
Classes:	N = 158		N = 216		N = 212		N = 213		N = 104	
Teachers:	N = 158		N = 171		N = 161		N = 145		N = 73	

Table 2

Organization of Content in Two Editions of the Programmed Course in Algebra for Teachers

Title		Content
1963	1964	
Unit 1	Introduction	Preview of course content, working instructions, etc.
Unit 2	Unit 1	Sets, Relations, Functions
Unit 3	Unit 2	Algebra of Real Numbers
	Unit 3	Algebraic Systems
	Test A	Test over Units 1-3
Unit 4	Unit 4	Order in the Real Number System
	Unit 5	Equations and Inequalities
	Unit 6	Absolute Value
	Unit 7	Completeness of the Real Number System
	Test B	Test over Units 4-7
Unit 5	Unit 8	Natural Numbers
	Unit 9	Integers
	Unit 10	Rational Numbers
Unit 6	Unit 11	Complex Numbers
	Test C	Test over Units 8-11
	Unit 12	Algebra of Real Functions
	Unit 13	Polynomials
Unit 7	Unit 14	Equivalence Relations and Groups
	Test D	Test over Units 12-14

Note: Each unit, in each edition, was followed by a unit test. Section Tests A, B, C, and D appeared only in the 1964 edition.

Table 3

Summary of Data from Which Posttest Regressions on STEP Pretest Were Calculated

Grade	Posttest	Years Given	Number of Students	Obtained Equation
9	STEP(2B)	1962,3,4,5	10,956	$\hat{y} = 9.2205 + .7598x$
9	MNL9(First version)	1965	2,873	$\hat{y} = 2.3431 + .2582x$
9	COOP(Number system)	1966	1,486	$\hat{y} = 6.1266 + .5976x$
9	COOP(Algebra I)	1966	1,486	$\hat{y} = 10.5519 + .5959x$
9	MNL9(Revised)	1966	1,486	$\hat{y} = 2.7625 + .2225x$
11	STEP(2B)	1962,3,4,5	4,476	$\hat{y} = 11.9206 + .7175x$
11	MNL11(First version)	1965	1,431	$\hat{y} = 1.3516 + .1484x$
11	COOP(Algebra II)	1966	736	$\hat{y} = 7.4232 + .5030x$
11	COOP(Algebra III)	1966	736	$\hat{y} = 2.4438 + .4410x$
11	MNL11(Revised)	1966	736	$\hat{y} = .3155 + .1942x$

Table 4

Relative Stability of Median Pretest Scores for Classes Taught by the Same Teachers in Consecutive Years

Group	Years of Consecutive Participation[1]	Number of Classes[2]	Rank Order Correlation or Average Correlation	Significance[3]
9C-2	1962,63	33	$r_s = .42$	$p < .02$
9C-3	1962,63,64	17	$r_{sav} = .42$	$p < .05$
9E-2	1962,63	24	$r_s = .46$	$p < .05$
9E-3	1962,63,64	10	$r_{sav} = .73$	$p < .01$
11C-2	1963,64	19	$r_s = .15$	n.s.
11C-3	1962,63,64	10	$r_{sav} = .13$	n.s.
11E-2	1963,64	19	$r_s = .73$	$p < .01$
11E-3	1962,63,64	5	$r_{sav} = .80$	$p < .01$

[1] Academic years designated by year of fall term.

[2] Within each group, the number of classes is equal to the number of teachers. The classes in 9C-3 form a subset of those in 9C-2, etc., so these groups are not independent. C and E curricula are not independent, since about half the teachers taught both a C and an E class. Grades are independent.

[3] Levels for r_{sav} from Ferguson (1965).

Table 5

Medians of Median Pretest Scores for Classes Taught by the Same Teachers in Consecutive Years

Group	Number of Classes[2]	Year[1] 1962	1963	1964
9C-2	33	23.50	22.00	
9C-3	17	23.50	22.00	23.50
9E-2	24	22.48	22.50	
9E-3	10	25.00	22.25	23.58
11C-2	19		32.00	30.67
11C-3	10	32.62	32.00	30.75
11E-2	19		33.33	32.50
11E-3	5	32.00	34.00	36.50

[1] Academic years designated by year of fall term.

[2] Within each group, the number of classes is equal to the number of teachers. The classes in 9C-3 form a subset of those in 9C-2, etc., so these groups are not independent. C and E curricula are not independent, since about half the teachers taught both a C and an E class. Grades are independent.

Table 6

Pretest Scores of Classes Taught by Teachers Who Did and Did Not Volunteer for the Algebra Course in 1963-64

		Teachers Who Volunteered			Teachers Who Did Not Volunteer		
Grade	Curriculum	Number of Classes	Median of Class Medians	Range of Class Medians	Number of Classes	Median of Class Medians	Range of Class Medians
9	C	25	21.33	15.62 to 27.20	8	24.00	19.67 to 27.00
9	E	47	23.25	14.67 to 33.62	19	24.00	17.00 to 28.50
11	C	12	32.12	29.50 to 43.70	7	32.12	31.33 to 34.50
11	E	22	33.29	22.75 to 41.00	13	33.50	28.75 to 40.75

Table 7

Pretest Scores of Classes Taught by Teachers Who Did and Did Not Return the First Unit in 1964-65

		Teachers Who Returned First Unit			Teachers Who Did Not Return First Unit		
Grade	Curriculum	Number of Classes	Median of Class Medians	Range of Class Medians	Number of Classes	Median of Class Medians	Range of Class Medians
9	C	7	21.75	15.75 to 23.00	15	19.83	13.70 to 29.12
9	E	12	22.87	13.25 to 27.00	22	21.59	13.00 to 33.75
11	C	10	30.96	25.50 to 38.30	5	27.50	24.00 to 29.25
11	E	16	33.50	20.00 to 40.00	9	31.17	24.00 to 44.00

Table 8

Correlations between Teachers' Scores and Median Pretest Scores of Their Classes in the Same Academic Year.

Year	Grade	Curriculum	Number of Classes	Rank Order Correlation
1963	9	C	7	-.37
	9	E	15	-.12
	11	C	See Note 1	
	11	E	9	.55
1964	9	C	7	.43
	9	E	15	.20
	11	C	10	.15
	11	E	15	.44

1 Insufficient number of classes for analysis.

Table 9

Correlations Between Teachers' Working Times and Median Pretest Scores of Their Classes in the Same Academic Year

Year	Grade	Curriculum	Number of Classes	Rank Order Correlation[1]
1963	9	C	7	.31
	9	E	15	.06
	11	C	See Note 2	
	11	E	9	-.20
1964	9	C	13	.39
	9	E	21	.21
	11	C	17	-.23
	11	E	24	-.16

[1] Shortest time and lowest score assigned lowest ranks.

[2] Insufficient number of classes for analysis.

Table 10

Relative Stability of Median Posttest Deviation Scores (STEP) for Classes Taught by the Same Teachers in Consecutive Years

Group	Years of Consecutive Participation[1]	Number of Classes[2]	Rank Order Correlation or Average Correlation	Significance
9C-2	1962,63	30	$r_s = .52$	$p < .01$
9C-3	1962,63,64	16	$r_{sav} = .26$	n.s.
9E-2	1962,63	23	$r_s = .18$	n.s.
9E-3	1962,63,64	10	$r_{sav} = .12$	n.s.
11C-2	1963,64	19	$r_s = .27$	n.s.
11C-3	1962,63,64	10	$r_{sav} = .31$	n.s.
11E-2	1963,64	18	$r_s = .15$	n.s.
11E-3	1962,63,64	5	$r_{sav} = -.40$	n.s.

[1] Academic years designated by year of fall term.

[2] Within each group, the number of classes is equal to the number of teachers. The classes in 9C-3 form a subset of those in 9C-2, etc., so these groups are not independent. C and E curricula are not independent, since about half the teachers taught both a C and an E class. Grades are independent.

Table 11

Medians of Median Posttest Deviation Scores (STEP) for Classes Taught by the Same Teachers in Consecutive Years.

Group	Number of Classes[2]		Year[1]		Significance of Trends[3]
		1962	1963	1964	
9C-2	30	1.60	-.30		p < .01
9C-3	16	1.60	.55	-.14	p < .01
9E-2	23	-.02	.18		n.s.
9E-3	10	-.28	.12	1.15	n.s. (p = .14)
11C-2	19		-.27	.71	n.s.
11C-3	10	.80	.00	1.00	n.s.
11E-2	18		-.53	.49	n.s.
11E-3	5	.62	.42	-.11	n.s.

[1] Academic years designated by year of fall term.

[2] Within each group, the number of classes is equal to the number of teachers. The classes in 9C-3 form a subset of those in 9C-2, etc., so these groups are not independent. C and E curricula are not independent, since about half the teachers taught both a C and an E class. Grades are independent.

[3] S-statistic ([redacted]on, 1965).

Table 12

Posttest Deviation Scores (STEP) of Classes Taught by Teachers Who Did and Did Not Volunteer for the Algebra Course in 1963-64

Grade	Curriculum	Teachers Who Volunteered			Teachers Who Did Not Volunteer		
		Number of Classes	Median of Class Medians	Range of Class Medians	Number of Classes	Median of Class Medians	Range of Class Medians
9	C	25	-.37[1]	-3.36 to 3.04	8	2.55[1]	-.97 to 5.54
9	E	47	.15	-12.44 to 3.88	19	.38	-5.64 to 2.89
11	C	12	.14	-2.80 to 2.22	7	.27	-2.67 to 1.78
11	E	22	.45	-2.30 to 3.64	13	.40	-5.29 to 2.96

[1] Difference between volunteers and non-volunteers significant at .01 level by the Mann-Whitney U test.

Table 13

Posttest Deviation Scores (STEP) of Classes Taught by Teachers Who Did and Did Not Return the First Unit in 1964-65

		Teachers Who Returned First Unit			Teachers Who Did Not Return First Unit		
Grade	Curriculum	Number of Classes	Median of Class Medians	Range of Class Medians	Number of Classes	Median of Class Medians	Range of Clas Medians
9	C	7	.68	-3.15 to 3.68	15	-.86	-2.46 t 4.91
9	E	12	.32	-3.50 to 10.71	22	-.58	-3.04 t 7.90
11	C	10	-.38	-5.72 to 5.28	5	5.69	-3.65 t 7.86
11	E	16	2.47	-9.44 to 9.73	9	.72	-5.97 t 5.69

Table 14

Correlations between Teachers' Scores and Median Posttest Deviation Scores (STEP) of Their Classes in the Same Academic Year

Year	Grade	Curriculum	Number of Classes	Rank Order Correlation
1963,64	9	C	7	-.26
	9	E	15	.10
	11	C	See Note 1	
	11	E	9	.00
1964,65	9	C	7	-.04
	9	E	15	.22
	11	C	10	.37
	11	E	15	-.16

[1] Insufficient number of classes for analysis.

Table 15

Correlations between Teachers' Working Times and Median Posttest Deviation Scores (STEP) of Their Classes in the Same Academic Year

Year	Grade	Curriculum	Number of Classes	Rank Order Correlation[1]
1963,64	9	C	7	-.22
	9	E	15	.00
	11	C	See Note 2	
	11	E	9	-.11
1964,65	9	C	13	-.11
	9	E	21	-.20
	11	C	17	-.32
	11	E	24	-.21

[1] Shortest time and lowest score assigned lowest ranks.

[2] Insufficient number of teachers for analysis.

Table 16

Relative Stability of Median Posttest Deviation Scores Obtained from MNL Tests for Classes Taught by the Same Teachers in 1964-65 and 1965-66

Group	Number of Classes	Rank Order Correlation	Significance
9C	16	.45	p < .05
9E	31	.43	p < .01
11C	7	.39	n.s.
11E	16	.56	p < .05

Table 17

Relative Stability of Median Posttest Deviation Scores Obtained from MNL Tests for Classes Taught by the Same Teachers in 1964-65 and 1965-66, within Experimental Curricula

Grade	Curriculum	Number of Classes	Rank Order Correlation
9	Ball State	8	.48
9	Illinios	8	.14
9	SMSG	12	.46
11	Ball State	9	.57
11	SMSG	7	.64

Table 18

Correlations of Median Posttest Deviation Scores Obtained from MNL Tests for Experimental and Conventional Classes Taught by the Same Teachers

Grade	Year	Number of Pairs of Classes	Rank Order Correlation	Significance
9	1964-65	17	.73	$p < .01$
9	1965-66	15	.57	$p < .05$
11	1964-65	5	-.20	n.s.
11	1965-66	5	.30	n.s.

Table 19

Posttest Deviation Scores (MNL) of Classes Taught by Teachers Who Did and Did Not Return the First Unit in 1964-65

		Teachers Who Returned First Unit			Teachers Who Did Not Return First Unit		
Grade	Curriculum	Number of Classes	Median of Class Medians	Range of Class Medians	Number of Classes	Median of Class Medians	Range of Class Medians
9	C	7	-1.15	-2.66 to .73	15	-.48	-3.91 to 2.89
9	E	12	-.20	-2.40 to 4.19	22	-.48	-1.69 to 3.05
11	C	9	-.65	-2.57 to 2.16	5	.31	-1.80 to 1.87
11	E	15	.00	-2.43 to 2.71	9	-.11	-1.57 to 1.76

Table 20

Correlations between Teachers' Scores in 1964-65 and Median Posttest Deviation Scores (MNL) of Their Classes in the Same Year

Grade	Curriculum	Number of Classes	Rank Order Correlation
9	C	7	.50
9	E	15	.21
11	C	9	.35
11	E	14	.34

Table 21

Correlations between Teachers' Working Times in 1964-65 and Median Posttest Deviation Scores (MNL) of Their Classes in the Same Year

Grade	Curriculum	Number of Classes	Rank Order Correlation[1]
9	C	13	-.04
9	E	21	-.06
11	C	15	-.24
11	E	22	.03

[1] Shortest time and lowest score assigned lowest ranks.

Table 22

Rank-Order Correlations between Teachers' Scores in 1964-65 and Median Posttest Deviation Scores of Their Classes in 1965-66

Grade	Curriculum	Number of Classes	Test		
			COOP Num. Sys.	COOP Alg. I	MNL 65
9	C	5	.38	-.18	.22
9	E	8	.10	.35	-.30
			COOP Alg. II	COOP Alg. III	MNL 65
11	C	See Note 1			
11	E	7	-.21	-.14	-.14

[1] Too few classes for analysis.

Table 23

Rank-Order Correlations between Teachers' Working Times in 1964-65 and Median Posttest Deviation Scores of Their Classes in 1965-66[1]

Grade	Curriculum	Number of Classes	Test		
			COOP Num. Sys.	COOP Alg. I	MNL 65
9	C	7	.06	-.15	-.58
9	E	12	-.12	-.16	-.63
			COOP Alg. II	COOP Alg. III	MNL 65
11	C	5	-.60	-.60	-.50
11	E	9	-.13	.05	-.15

[1] Shortest time and lowest score assigned lowest ranks.

Table 24

Correlations between Teachers' Scores in 1964-65 and Median Posttest Deviation Scores (MNL) of Upper and Lower Ability Students in Their Classes in the Same Year

Grade	Curriculum	Number of Classes	Rank-Order Correlations: Students below Pretest Median of Class	Rank-Order Correlations: Students at or above Pretest Median of Class
9	C	7	.46	.50
9	E	15	.34	.09
11	C	9	.90	.13
11	E	13	.43	.43

Table 25

Correlations between Teachers' Working Times in 1964-65 and Median Posttest Deviation Scores (MNL) of Upper and Lower Ability Students in Their Classes in the Same Year

Grade	Curriculum	Number of Classes	Correlations[1]: Students below Pretest Median of Class	Correlations[1]: Students at or above Pretest Median of Class
9	C	13	-.12	.09
9	E	21	-.14	.10
11	C	15	-.42	-.17
11	E	21	-.13	.03

[1] Shortest time and lowest score assigned lowest ranks.

Table 26

Numbers of Tenth-grade Teachers Returning Each Unit of the Programmed Correspondence Course in Geometry, November, 1965 to April, 1967 [1]

Unit	Number of Teachers
1	74
2	35
3	13
4	10
5	8
6	8
7	7
8	7
9	6
10	5
11	5

[1] The course was originally sent to 260 teachers.

Appendix A: Classes taught by 9th and 11th grade teachers during each year of their participation in the Five-State Project.

Note: C = Conventional class; E = Experimental class. Key to Experimental Curricula:

A - Ball State Teachers College Experimental Mathematics Program

B - University of Illinois Committee on School Mathematics

C - University of Maryland Mathematics Project

D - School Mathematics Study Group

Key to school codes follows on A-9.

Classes for which adequate test data were not obtained have been omitted from the tables.

Grade 9

Teacher Code	School	Experimental Curriculum	Academic Year 1961-62	1962-63	1963-64	1964-65	1965-66
224	114	A		E	E	E	
225	098	A			E	E	
229	001	A	C	E		E	E
230	163	A	C	E			
231	099	A	C	CE		CE	
264	151	A	C	CE	E	E	
324	217	A	C	E		CE	
334	225	A	C	CE			
335	226	A	C	E			
336	156	A	C	E			
343	232	A	C	E			
344	209	A	C	CE			
346	162	A	C	E			
355	242	A	C	E	E		
357	182	A	C	CE			
362	245	A			E		
363	246	A				CE	CE
384	260	A	C	CE	CE	CE	
389	265	A	C	E	E		
400	276	A	C	CE			
426	246	A	C	CE	CE		
428	298	A	C	E	E		
440	308	A			E		
504	350	A	C	E			
505	351	A	C	CE	CE	CE	E
591	406	A		C	E	E	
608	412	A		C	CE	CE	
615	374	A		C	E		
618	030	A		C	E		
619	417	A		C	E		
629	398	A		C	E		
632	400	A		C	E	E	E
644	418	A		C	CE		
680	431	A		C	CE	CE	
693	445	A				E	
694	446	A		C	CE	CE	CE
722	463	A		C	CE		
729	459	A		C	CE		
734	467	A		C	CE	CE	CE
767	486	A			C	CE	CE
772	488	A			C	E	
797	522	A			C	CE	
831	148	A			C	E	E
839	510	A			C	E	
850	498	A			C	E	

Grade 9 (cont.)

Teacher Code	School	Experimental Curriculum	Academic Year 1961-62	1962-63	1963-64	1964-65	1965-66
876	585	A				C	CE
920	576	A				E	E
930	580	A				E	
933	578	A				CE	
226	043	B	C	E	E	E	E
227	155	B	C	CE	CE	CE	
228	165	B	C	CE			
261	147	B	C	CE	CE	CE	CE
278	183	B		CE			CE
325	206	B	C	CE			
328	220	B	C	CE	CE		
337	227	B	C	E	E	E	E
340	229	B	C	CE			
341	230	B	C	CE	CE		
348	235	B	C	E			
358	243	B	C	CE	CE		
365	248	B	C	E	E		
382	258	B	C	E	E		
386	262	B	C	E			
410	285	B	C	CE			CE
416	290	B		E			
425	263	B	C	CE			
639	047	B		C	CE	CE	CE
656	393	B		C	E		
675	436	B		C	E		
676	207	B		C	E	E	
683	440	B		C	E		
701	449	B		C	E		
718	461	B		C	CE	CE	
730	466	B		C	CE	CE	
737	469	B		C	CE		
739	471	B		C	CE	CE	CE
740	472	B		C	CE		
768	474	B			C	CE	
794	520	B			C	CE	
813	535	B				E	
815	537	B			C	E	E
819	541	B			C	CE	
830	550	B			C	E	E
890	474	B				C	CE
894	595	B				C	CE
896	561	B				E	E
900	563	B				CE	CE
906	568	B				CE	
628	098	C				E	E

Grade 9 (cont.)

Teacher Code	School	Experimental Curriculum	Academic Year 1961-62	1962-63	1963-64	1964-65	1965-66
202	138	D		E	E	E	
232	161	D	C	E			
234	129	D	C	E			
235	159	D	C	E			
240	176	D		CE	CE		
255	038	D	C	CE	CE		
262	148	D	C	E			
263	149	D	C	E	E	E	
265	153	D	C	CE	CE		
282	187	D				E	CE
300	189	D		E	E		
304	202	D		E			
310	184	D		CE			
326	218	D	C	CE	CE	CE	
327	219	D	C	CE			
331	223	D	C	CE	CE		
332	224	D	C	CE	CE	CE	CE
333	202	D	C	E			
338	172	D	C	CE			
347	234	D	C	CE	E		
350	237	D	C	CE	CE	CE	E
359	198	D	C	CE			
360	224	D	C	CE			
377	247	D	C	CE			
381	257	D	C	E			
385	261	D	C	E			
390	266	D		E	E		
397	273	D		E			
406	281	D	C	E	E		
431	301	D		E			E
438	255	D				E	
439	307	D		E			
600	409	D				E	
609	413	D				CE	
617	416	D		C	E		
635	417	D				CE	CE
648	411	D		C	CE		
655	424	D		C	CE		
666	345	D		C	E		
667	430	D		C	CE		
677	437	D		C	CE		
685	442	D		C	CE	CE	
686	443	D				E	
687	429	D		C	CE	CE	
703	431	D		C	CE	CE	

Grade 9 (cont.)

Teacher Code	School	Experimental Curriculum	Academic Year				
			1961-62	1962-63	1963-64	1964-65	1965-66
710	454	D				E	
711	455	D		C	CE		
735	468	D			E		
758	480	D		C	E		
761	482	D		C	E	E	E
762	178	D			C	CE	
769	130	D				CE	CE
773	490	D				CE	CE
777	493	D			C	CE	CE
793	227	D			C	E	
800	525	D			C	CE	CE
803	527	D			C	CE	CE
818	540	D				E	
834	552	D			C	E	E
835	553	D			C	CE	E
849	487	D			C	CE	
853	500	D			C	CE	
858	502	D			C	E	
863	505	D			C	CE	
865	500	D			C	E	
885	591	D				C	CE
915	216	D				CE	
917	574	D				E	E
929	480	D				E	E
866	274	-			C		
875	584	-				C	
886	587	-				C	
892	593	-				C	
895	581	-				C	

Grade 11

Teacher Code	School	Experimental Curriculum	Academic Year 1961-62	1962-63	1963-64	1964-65	1965-66
236	179	A		CE	CE	CE	CE
238	166	A			C		C
245	174	A	C	E	E	E	E
281	186	A			E		E
290	195	A	C	CE			
292	197	A	C	CE	CE		
336	156	A				E	
378	254	A	C	E	C		
391	267	A		E			
392	268	A	C	E	E		
419	293	A			CE	CE	CE
423	263	A	C	CE	CE		
452	317	A			E	E	E
568	391	A		C	E	E	E
576	397	A		C	E	E	
587	404	A				CE	
593	404	A		C	CE	CE	
659	425	A		C	CE		
671	434	A		C	CE	CE	CE
716	459	A		C	CE		
741	472	A				CE	CE
748	474	A		C	CE	CE	
749	475	A		C	CE	E	E
764	064	A			C	CE	
789	516	A				CE	
791	518	A			C		
821	542	A			C	CE	
824	545	A			C	E	
841	512	A			C	CE	CE
855	501	A			C	CE	
870	582	A				C	E
897	497	A				E	E
899	038	A				CE	
907	569	A				CE	CE
916	573	A				CE	
926	577	A				E	
927	578	A				CE	
931	580	A				E	
232	161	D				E	E
235	159	D				E	
240	176	D	C				
244	175	D	C	CE	E	E	E
271	164	D			E		
291	196	D	C	CE			
293	189	D	C	CE			

Grade 11 (cont.)

Teacher Code	School	Experimental Curriculum	Academic Year 1961-62	1962-63	1963-64	1964-65	1965-66
295	198	D	C	E	E		
296	199	D	C	E	E	E	
306	204	D			CE		
310	184	D	C				
311	207	D		E	E		
371	203	D	C	E			
374	236	D	C	CE			
375	253	D		CE	CE	CE	
379	255	D		CE			
394	270	D	C	E			
395	271	D		E			
398	274	D				E	E
429	299	D	C				
458	320	D		E	E	E	E
481	334	D	C				
557	172	D		CE	CE		E
582	401	D				CE	
583	150	D		C	CE	CE	
590	400	D		C	E	E	
612	150	D		C	CE	CE	
614	007	D		C	E		
640	400	D		C	E		
691	442	D		C	CE	CE	
699	448	D		C	CE	CE	
702	425	D		C	CE		
745	299	D		C	CE	CE	
759	477	D		C	CE	CE	
769	130	D			C		
778	174	D			C		
781	492	D			C	CE	
782	495	D			C	E	E
802	526	D			C	E	E
810	533	D			C	E	
811	534	D				E	
812	516	D				E	
818	540	D			C		
825	546	D			C	E	
836	508	D				E	
840	511	D			C	E	E
845	479	D			C		
854	500	D			C	CE	
862	504	D			C	CE	CE
874	496	D				C	E
879	587	D				C	CE

Grade 11 (cont.)

Teacher Code	School	Experimental Curriculum	Academic Year				
			1961-62	1962-63	1963-64	1964-65	1965-66
884	587	D				C	CE
902	565	D				CE	
903	566	D				E	E
910	502	D				CE	
928	579	D				C	
832	543	-			C		

School Code Number	School	Town	State
1	Academy of Holy Angels	Minneapolis	Minn.
7	Aurora-Hoyt Lakes H.S.	Aurora	Minn.
30	Cook County High School	Grand Marais	Minn.
38	Edina-Morningside Sr. H.S.	Edina	Minn.
43	Faribault Junior H.S.	Faribault	Minn.
47	Glenwood High School	Glenwood	Minn.
64	Lakeville High School	Lakeville	Minn.
76	Morris Junior H.S.	Morris	Minn.
98	C. Sandberg Jr. H.S.	Minneapolis	Minn.
99	Sebeka High School	Sebeka	Minn.
114	Stillwater Jr. H.S.	Stillwater	Minn.
129	Wayzata High School	Wayzata	Minn.
130	West Senior H.S.	Minneapolis	Minn.
138	Wells Senior High School	Wells	Minn.
147	Manson Community H.S.	Manson	Iowa
148	L.D.F. Community H.S.	LeGrand	Iowa
149	Aurelia Community H.S.	Aurelia	Iowa
150	Richfield Senior H.S.	Richfield	Minn.
151	Jefferson Community H.S.	Jefferson	Iowa
153	Bedford Community H.S.	Bedford	Iowa
155	Richfield West Jr. H.S.	Richfield	Minn.
156	Schleswig Community H.S.	Schleswig	Iowa
159	Pine River H.S.	Pine River	Minn.
161	Marian High School	Marian	Minn.
162	Mount Ayr Community H.S.	Mount Ayr	Iowa
163	Herman High School	Herman	Minn.
164	Graettinger Community H.S.	Graettinger	Iowa
165	South High School	Minneapolis	Minn.
166	St. Adrian High School	St. Adrian	Minn.
172	Shenandoah Community H.S.	Shenandoah	Iowa
174	Columbia Heights H.S.	Minneapolis	Minn.
175	Robbinsdale High School	Robbinsdale	Minn.
176	Sandstone High School	Sandstone	Minn.
178	Barnesville High School	Barnesville	Minn.
179	Aitkin High School	Aitkin	Minn.
182	Bennett Community H.S.	Bennett	Iowa
183	Lynnville-Sully Community H.S.	Sully	Iowa
184	West High School	Davenport	Iowa
186	Columbus Community H.S.	Columbus Jct.	Iowa
187	Lenox Community H.S.	Lenox	Iowa
189	Clinton Community H.S.	Clinton	Iowa
195	New Hampton Community H.S.	New Hampton	Iowa
196	Roosevelt High School	Des Moines	Iowa
197	Lincoln High School	Des Moines	Iowa
198	Maurice-Orange City Comm. H.S.	Orange City	Iowa
199	Lake View-Auburn Community H.S.	Lake View	Iowa
202	Watertown Junior High School	Watertown	S.D.
203	Watertown Senior High School	Watertown	S.D.
204	Central High School	Aberdeen	S.D.

School Code Number	School	Town	State
206	Sheldon Community High School	Sheldon	Iowa
207	Burt Community High School	Burt	Iowa
209	Boone Community High School	Boone	Iowa
216	Melcher-Dallas Community H.S.	Melcher	Iowa
217	Moravia Community H.S.	Moravia	Iowa
218	Yankton Junior High School	Yankton	S.D.
219	Sudlow Junior High School	Sudlow	Iowa
220	Gladbrook Community High School	Gladbrook	Iowa
223	Centerville Ind. High School	Centerville	Iowa
224	Monroe Junior High School	Aberdeen	S.D.
225	North High School	Des Moines	Iowa
226	Primghar Community High School	Primghar	Iowa
227	Spirit Lake Community High School	Spirit Lake	Iowa
229	Dows Community High School	Dows	Iowa
230	Sumner Community High School	Sumner	Iowa
232	So. Winneshiek Community H.S.	So. Winneshiek	Iowa
234	State Center Community H.S.	West Marshall	Iowa
235	Fonda Community High School	Fonda	Iowa
236	Osage Community High School	Osage	Iowa
237	Brookings Junior High School	Brookings	S.D.
242	Oakland Ind. School	Oakland	Iowa
243	Franklin Junior High School	Franklin	Iowa
244	Traer-Clutier Community H.S.	Traer	Iowa
245	South Junior H.S.	Grand Forks	N.D.
246	Williston Junior High School	Williston	N.D.
247	Jamestown Junior High School	Jamestown	N.D.
248	Janesville High School	Janesville	Iowa
253	Brookings High School	Brookings	S.D.
254	Central High School	Grand Forks	N.D.
255	Mayville High School	Mayville	Wisc.
257	Reeder High School	Reeder	N.D.
258	Goodrich High School	Goodrich	N.D.
260	Wahpeton High School	Wahpeton	N.D.
261	West Fargo High School	Fargo	N.D.
262	Trenton High School	Trenton	N.D.
263	Central High School	Devils Lake	N.D.
265	Valley Junior High School	Grand Forks	N.D.
266	Waupaca High School	Waupaca	Wisc.
267	Elk Mound High School	Elk Mound	Wisc.
268	Wilmot High School	Wilmot	Wisc.
270	Sun Prairie High School	Sun Prairie	Wisc.
271	Owen-Withee Senior High School	Owen	Wisc.
273	Ashland-Ondossagon High School	Ashland	Wisc.
274	Edgerton High School	Edgerton	Wisc.
276	Webb High School	Reedsburg	Wisc.
281	Onalaska High School	Onalaska	Wisc.
285	Central High School	Superior	Wisc.
290	Turtle Lake High School	Turtle Lake	Wisc.
293	Greendale High School	Greendale	Wisc.

School Code Number	School	Town	State
298	Hazen High School	Hazen	N.D.
299	Jamestown High School	Jamestown	N.D.
301	Glenfield High School	Glenfield	N.D.
307	South High School	Sheboygan	Wisc.
308	Ashland Junior High School	Ashland	Wisc.
317	Alma High School	Alma	Wisc.
320	Unity High School	Milltown	Wisc.
334	Beresford High School	Beresford	S.D.
345	Oskaloosa Junior H.S.	Oskaloosa	Iowa
350	Emmetsburg Community School	Emmetsburg	Iowa
351	Wilton Community H.S.	Wilton Jct.	Iowa
374	Arch. Murray Memorial H.S.	St. Paul	Minn.
391	Lake Park High School	Lake Park	Minn.
393	Mabel High School	Mabel	Minn.
397	Falls High School	International Falls	Minn.
398	Clara City Public School	Clara City	Minn.
400	Lincoln High School	Thief River Falls	Minn.
401	Luverne High School	Luverne	Minn.
404	Fairmont High School	Fairmont	Minn.
406	Wabasso High School	Wabasso	Minn.
409	Trimont Senior High School	Trimont	Minn.
411	Chisholm Junior High School	Chisholm	Minn.
412	Mahnomen High School	Mahnomen	Minn.
413	Fridley High School	Minneapolis	Minn.
416	Excelsior Jr. High School	Excelsior	Minn.
417	Eveleth Jr. High School	Eveleth	Minn.
418	Backus Junior High School	International Falls	Minn.
424	North Branch H.S.	North Branch	Minn.
425	Ames Senior High School	Ames	Iowa
429	Iowa Falls Comm. Jr. School	Iowa Falls	Iowa
430	Paullina Community School	Paullina	Iowa
431	Central Junior High School	Ames	Iowa
434	Sac Community High School	Sac City	Iowa
436	Mallard Community School	Mallard	Iowa
437	LeMars Community School	LeMars	Iowa
440	Collins Community School	Collins	Iowa
442	Washington High School	Cherokee	Iowa
443	St. Boniface High School	New Vienna	Iowa
445	Hamburg Community School	Hamburg	Iowa
446	Murray Community School	Murray	Iowa
448	Iowa Falls Sr. High School	Iowa Falls	Iowa
449	Gilmore City-Bradgate Comm. Sch.	Gilmore City	Iowa
454	Monroe Senior High School	Monroe	Wisc.
455	Washington High School	Manitowoc	Wisc.
459	West Bend High School	West Bend	Wisc.
461	Merrill Junior High School	Merrill	Wisc.
463	Mukwonago Union H.S.	Mukwonago	Wisc.
466	Algoma High School	Algoma	Wisc.
467	Neilsville High School	Neilsville	Wisc.

School Code Number	School	Town	State
468	Turtle Mt. Comm. School	Belcourt	N.D.
469	Mandan Junior H.S.	Mandan	N.D.
471	Kenmare High School	Kenmare	N.D.
472	Bishop Ryan High School	Minot	N.D.
474	Valley City High School	Valley City	N.D.
475	Central High School	Grafton	N.D.
477	Rapid City High School	Rapid City	S.D.
479	St. Mary High School	Dell Rapids	S.D.
480	O'Gorman High School	Sioux Falls	S.D.
482	Washington Junior H.S.	Rapid City	S.D.
486	Johanna Junior H.S.	New Brighton	Minn.
487	Lincoln High School	Alma Center	Wisc.
488	St. Anthony Village H.S.	Minneapolis	Minn.
490	Sanford Junior High School	Minneapolis	Minn.
492	St. Francis High School	St. Francis	Minn.
493	Columbia Heights Jr. H.S.	Columbia Hts.	Minn.
495	Wheaton High School	Wheaton	Minn.
496	Greenway High School	Coleraine	Minn.
497	Southwest High School	Minneapolis	Minn.
500	Pulaski High School	Pulaski	Wisc.
501	Valders High School	Valders	Wisc.
502	Beloit Memorial High School	Beloit	Wisc.
504	Ripon Senior High School	Ripon	Wisc.
505	Cadott High School	Cadott	Wisc.
508	St. Mary High School	Richardton	N.D.
510	Leeds High School	Leeds	N.D.
511	Watford City High School	Watford	N.D.
512	Napoleon High School	Napoleon	N.D.
516	Anamosa Community School	Anamosa	Iowa
518	Ocheyedan Community School	Ocheyedan	Iowa
520	Perry Community School	Perry	Iowa
522	Marcus Community School	Marcus	Iowa
525	Central City Comm. School	Central City	Iowa
526	Farragut Community School	Farragut	Iowa
527	Northwood-Kensett Comm. Sch.	Northwood	Iowa
533	Monticello Community School	Monticello	Iowa
534	Orange-Township Cons. School	Waterloo	Iowa
535	Garrison Cons. School	Garrison	Iowa
537	Carson-Macedonia Comm. Sch.	Carson	Iowa
540	Correctionville Ind. School	Correctionville	Iowa
541	Adel Community School	Adel	Iowa
542	Atlantic Community School	Atlantic	Iowa
543	South Hamilton Comm. School	Jewell	Iowa
545	Sergeant Bluff-Luten Comm. Sch.	Sergeant Bluff	Iowa
546	Durant Community School	Durant	Iowa
550	Lincoln Central Comm. School	Gruver	Iowa
552	Prairie City Community School	Prairie City	Iowa
553	Milford Community School	Milford	Iowa
561	Lutheran High School	Mayer	Minn.
563	Bemidji Jr. High School	Bemidji	Minn.

School Code Number	School	Town	State
565	Canby High School	Canby	Minn.
566	Bagley High School	Bagley	Minn.
568	East Troy Comm. H.S.	East Troy	Wisc.
569	St. Francis High School	St. Francis	Wisc.
573	Postville Comm. H.S.	Postville	Iowa
574	Lawton High School	Lawton	Iowa
576	Shellsburgh High School	Shellsburgh	Iowa
577	Tioga High School	Tioga	N.D.
578	Miller High School	Miller	S.D.
579	Spearfish High School	Spearfish	S.D.
580	Belle Fourche H.S.	Belle Fourche	S.D.
581	Gettysburg Junior H.S.	Gettysburg	S.D.
582	Little Falls High School	Little Falls	Minn.
584	Schiocton High School	Schiocton	Wisc.
585	Oakfield High School	Oakfield	Wisc.
587	Marshalltown High School	Marshalltown	Iowa
591	North Junior High School	Fort Dodge	Iowa
593	New Salem High School	New Salem	N.D.
595	Kimball High School	Kimball	S.D.

Appendix B: Minnesota National Laboratory Mathematics Tests for grades nine and eleven.

1964 Edition	Page B-2
1965 Edition	Page B-39

Minnesota National Laboratory
State Department of Education
Mathematics Section

Code: XXXVIII--B--336

Minnesota National Laboratory Mathematics Tests

(1964 Edition)

INSTRUCTIONS FOR ADMINISTRATION

NOTE: Please read completely before the period scheduled to administer these tests.

GENERAL INFORMATION:

There are several forms of the Minnesota National Laboratory Mathematics Tests at each of several grade levels as indicated below.

GRADE	LEVEL	FORM
7 & 8	1	A & B
9	2	C & D
10	3	E & F
11 & 12	4	J, K, & L

These tests have been constructed to cover the mathematics content typically included at each indicated grade level. The objective in this regard has been to select test items covering the general and most central principles and concepts introduced at a given grade level which are common to both conventional and modern mathematics programs. This means that the content of test forms at one grade level will not overlap with the content of forms for a lower grade level.

Because the tests are focused on the content at a given grade level, pupils are likely to experience some difficulty with the test prior to having been exposed to the content covered in that grade. This will probably lead to expressions of unfamilarity and difficulty by pupils after taking the pretest forms in the fall, which, however, will not be the case for the forms to be administered in the spring.

ADMINISTRATION PROCEDURES: (Please read before the period scheduled for the test)

1. Pass out the testing materials. Each pupil should receive one copy of the test, one Mark Sense Answer Card, one special electrographic pencil and several pieces of scratch paper. (The latter is quite important so the pupil will not find it necessary to write on the test booklet itself)

2. Instruct pupils to print their name only in the space provided on the Mark Sense Answer Card. Pupils are not to put their names on the test booklet and they are not to put anything other than their names and answers on the Answer Card.

3. Instruct the pupils to read the following test instructions to themselves while you read them aloud. Following these instructions are a number of specific points on questions that pupils might ask. You should read these (to yourself) prior to administering the test.

GENERAL INSTRUCTIONS:

This is a test of how well you understand and can apply what you have learned about mathematics. You are to read and try to answer each of these questions even though some of the questions may be about things you have not yet studied.

Do not make any marks on the test booklet itself.

You are to indicate your answer only on the Answer Card using the special electrographic pencil you have been given.

Do your figuring only on the scratch paper you have been given.

First, print your name in the space provided on the Answer Card. Now, read carefully the test procedures given below.

Test Procedures:

1. Read each item carefully and completely, think about the problem and do any figuring you want on the scratch paper.

2. Read the alternative answers carefully and select the one correct answer for each question.

3. Indicate your answer on the Answer Card by blackening the space over and around the letter (A, B, C, etc.) of the answer you have chosen, using the electrographic pencil. Be sure that the row in which you mark the answer has the same number as the question. If you wish to change an answer, be sure to erase the previous answer completely.

4. Since your score depends upon the number of correct answers that you mark, you should try to answer each question. However, do not spend too much time on any one item. If you have difficulty determining the correct answer, make the most careful guess you can and go on to the next item. If you finish before time is called, you can go back and work the items that were more difficult. Try to do as well as you can.

If you have any questions about what you are to do, ask your teacher now.

Do not begin working until the teacher tells you to start.

As soon as the questions have been answered the pupils may begin. They should be allowed 45 minutes from the time they begin to complete the test.

Answer any questions the pupils have concerning the procedures in terms of the information given in the instructions or in the supplementary information given below.

SUPPLEMENTARY INFORMATION FOR TEST ADMINISTRATORS:

1. Since the pupils score is the number of correct answers, there is no penalty for guessing. Pupils should be cautioned to guess carefully, however.

2. If pupils finish before time is called they should cover their Answer Cards with the test booklets and remain quiet.

3. Time should be called 45 minutes after the signal to begin was given.

4. The Answer Cards and test booklets should be collected separately and the Answer Cards from each class should be kept separate.

5. It is suggested that following the test you point out that many of the test items were difficult because they concerned topics and materials the pupils have not yet studied. You might further point out, such a test is necessary to provide an indication of how much is learned when a test like this is given at the end of the semester. These comments should be made only after the test has been administered.

6. Please return all copies of the test booklets because these booklets will be reused for other classes for the spring testing.

If there are any questions as to procedures for administration of this test please contact:

Dr. James J. Ryan
Minnesota National Laboratory
State Department of Education
St. Paul 1, Minnesota
Telephone: 373-4011

PLEASE NOTE: This set of test booklets is to be used for each of the classes participating in the Evaluation of Secondary Mathematics Curricula Program.

Code: XXXVIII-B-328

MINNESOTA NATIONAL LABORATORY

MATHEMATICS TEST

LEVEL TWO FORM C

(Grade Nine, 1964 Edition)

General Instructions:

This is a test of how well you understand and can apply what you have learned about mathematics. You are to read and try to answer each of these questions, even though some of the questions may be about things you have not yet studied.

Do not make any marks on the test booklet itself.

You are to indicate your answer only on the Answer Card using the special electrographic pencil you have been given.

Do your figuring only on the scratch paper you have been given.

First, print your name in the space provided on the Answer Card.

Now, read carefully the test procedures given below.

Test Procedures:

1. Read each item carefully and completely, think about the problem and do any figuring you want on the scratch paper.

2. Read the alternative answers carefully and select the one correct answer for each question.

3. Indicate your answer on the Answer Card by blackening the space over and around the letter (A, B, C. etc.) of the answer you have chosen, using the electrographic pencil. Be sure that the row in which you mark the answer has the same number as the question. If you wish to change an answer, be sure to erase the previous answer completely.

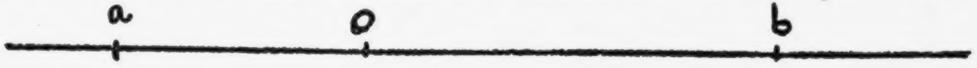
4. Since your score depends upon the number of correct answers that you mark, you should try to answer each question. However, do not spend too much time on any one item. If you have difficulty determining the correct answer, make the most careful guess you can and go on to the next item. If you finish before time is called, you can go back and work on the items that were more difficult. Try to do as well as you can.

If you have any questions about what you are to do, ask your teacher now.

Do not begin working until the teacher tells you to start.

C

1. Given that a and b are the coordinates of two points on the number line, which one of the following expressions gives the coordinates of the point midway between them?



A. $\sqrt{b - a}$ B. $\frac{a + b}{2}$ C. $\frac{b + 2a}{2}$ D. $\sqrt{a^2 + b^2}$ E. $\frac{b - a}{2}$

2. Which one of the following is equivalent to the expression $4(k + 4)3$?

A. $4k + 7$ B. $4k + 12$ C. $7k + 28$ D. $4(k + 4) \cdot (k + 4)3$ E. $12k + 48$

3. If the area of a triangle is 48 and the altitude is less than or equal to 12, which one of the following is true of the base?

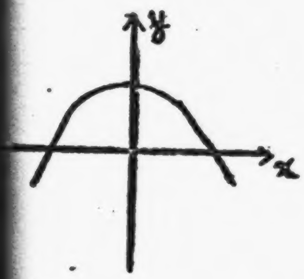
A. The base is greater than or equal to 4.
B. The base is greater than 4 and less than 8.
C. The base is greater than or equal to 8.
D. The base is less than or equal to 4.
E. The base is less than or equal to 8.

4. If y is an odd number, what is true of the expression $y^2 + ay$?

A. It is odd if a is even.
B. It is even if a is even.
C. It is odd if a is odd.
D. It is always even.
E. It is always odd.

5. Which one of the following is not the graph of a function $y = f(x)$?

A.



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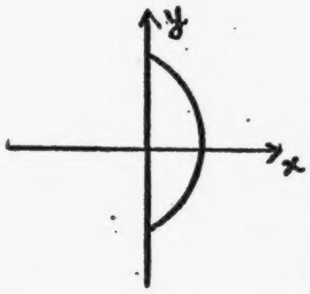

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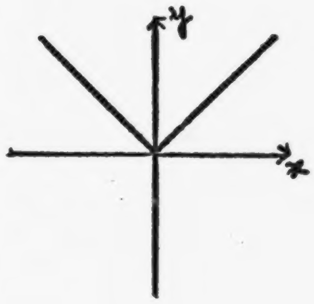

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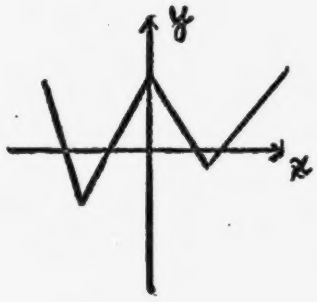
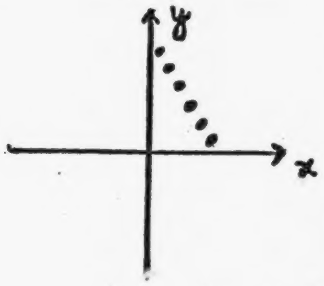
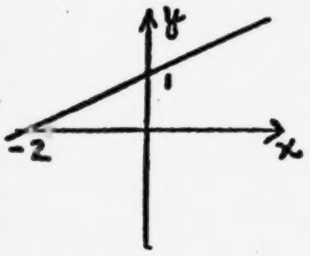


E.

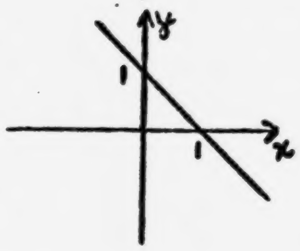


6. Which one of the following represents the graph of the equation $y = x + 1$?

A.



B.



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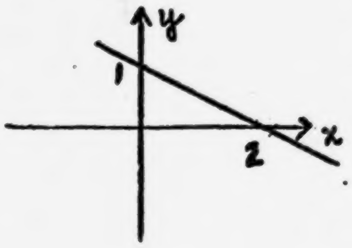
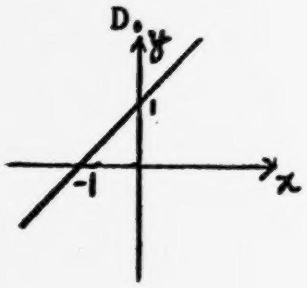
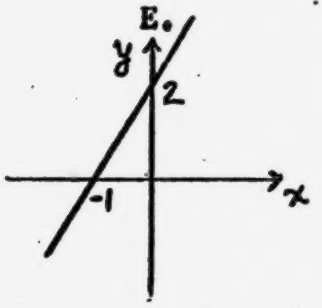


D.



E.



7. Which one of the following is true?

A. $4^5 \cdot 5^4 = 20^9$ B. $4^4 + 4^4 = 4^8$ C. $(4^4)^2 = 4^{16}$ D. $4^4 \cdot 5^4 = 20^4$

E. $4^4 \cdot 4^4 = 16^8$

8. Consider the following table defining the operation Δ .

Which one of the axioms below does not apply to the system?

Δ	α	β
α	α	β
β	α	β

A. Commutative law.
B. Closure principle.
C. Associative law.
D. Uniqueness principle.
E. Transitive property.

9. Given $F = \frac{a}{b} + \frac{c}{d}$ and $d = kb$, which one of the expressions below is equivalent to F ?

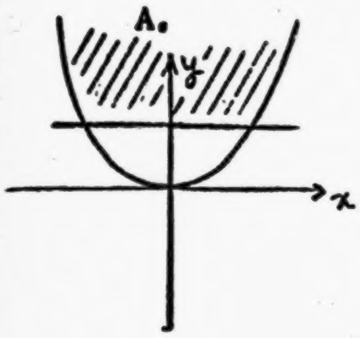
A. $\frac{a + kc}{d}$ B. $\frac{k(a + c)}{d}$ C. $\frac{ka + c}{b}$ D. $\frac{a + kc}{b}$ E. $\frac{ka + c}{kb}$

10. Which one of the following is the graph of the intersection of the following system of equations:

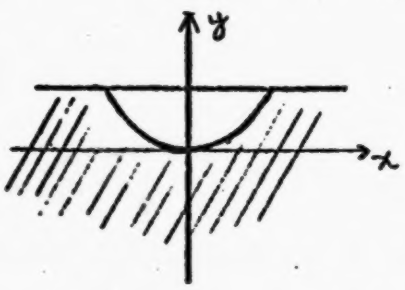
$$y \geq x^2$$

$$y \leq 2$$

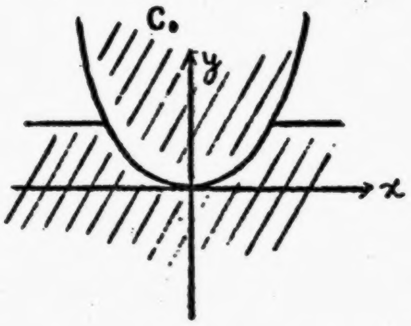
A.



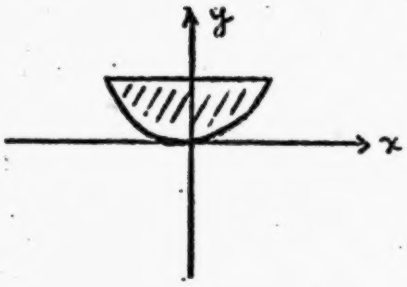
B.



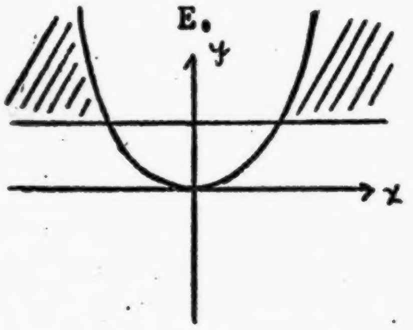
C.



D.



E.



11. The sum $a + b$ is smaller than a and also smaller than b. Which one of the following statements is correct?

A. Both a and b are positive.
B. Both a and b are negative.
C. a is positive and b is negative.
D. a is negative and b is positive.
E. One of the numbers is zero.

12. The larger of two numbers is 17 more than twice the smaller number. If the larger number is divided by the smaller number, the quotient is 3, with a remainder of 7. If x is the smaller number, which one of the following equations should be used to determine x ?

A. $2x - 17 = 10x$
B. $2x + 17 = 3x + 7$
C. $\frac{2x + 17}{3x} = 7$
D. $2x + 17 = 3x^2 + 7x$
E. $2x - 17 = 10$

13. Consider the following two statements about a 2-digit number we wish to find.

    1. The product of the digits is 14 and their sum is 9.
    2. If the number were doubled, the resulting number would be a 3-digit number.

    Which of the statements above must be used to solve the problem?

    A. Only statement 1.
    B. Only statement 2.
    C. Both statement 1 and statement 2.
    D. Either statement 1 or statement 2.
    E. More information is needed to find the number.

14. If $(x)(x - 4) = -3$, which one of the following pairs of substitutions for x makes the equation true?

    A. $x = -3$ or $x = 1$ B. $x = 0$ or $x = 4$ C. $x = -3$ or $x = 4$

    D. $x = 0$ or $x = 1$ E. $x = 3$ or $x = 1$

15. Define, for any two numbers a and b

    $$a \Delta b = \text{least common multiple of } a \text{ and } b,$$

    $$a \theta b = \text{greatest common divisor of } a \text{ and } b.$$

    What is the value of $(3 \Delta 5) \theta (3 \theta 12)$?

    A. 3 B. 5 C. 12 D. 15 E. 60

16. If $|x - y| = x$ and $y \neq 0$, what is the value of y ?

    A. $2x$ B. x C. $-2x$ D. $x - |x|$ E. $\pm x$

17. Consider the following proof:

    $$a^2 = b^2$$
    $$a^2 - b^2 = 0$$
    $$(a - b)(a + b) = 0$$
    $$a - b = 0 \quad \text{or} \quad a + b = 0$$
    $$a = b \quad \text{or} \quad a = -b$$

    The proof shows that:

    A. A difference of squares, $a^2 - b^2$, is factorable as $(a + b)(a - b)$.
    B. $a^2 = b^2$ and $a = b$ are not equivalent sentences.
    C. If the product of two factors is zero, one of the factors must be equal to zero.
    D. If $a^2 = b^2$, then $a = b$.
    E. If $a = b$, then $a^2 = b^2$.

18. What value or values of b will make $x = 1$ a solution of the following equation?

$$x^2 + 2b^2x - 5b + 1 = 0$$

A. Only $\frac{1}{2}$ B. Only 0 C. $\frac{1}{2}$ and 2 D. 0 and 2 E. Only 1

19. The operation Δ is defined for ordered pairs of integers as follows:

$(a,b) \Delta (c,d) = (a - c, b + d)$ where $+$ and $-$ are the usual operations of addition and subtraction.

Why is the operation Δ not commutative?

A. Subtraction is not commutative.
B. No operation concerning integers is commutative.
C. The system is closed.
D. Commutativity involves only two elements.
E. $b + d = d + b$

20. For what values of x is the following inequality true?

$$x^2 + 1 < 2x + 1$$

A. $x > 0$ B. $0 < x$ or $x > 2$ C. $x > 2$ D. $x < 2$ E. $0 < x < 2$

21. What is the smallest integer x which satisfies the following inequality?

$$\frac{5}{7} > \frac{x}{-14}$$

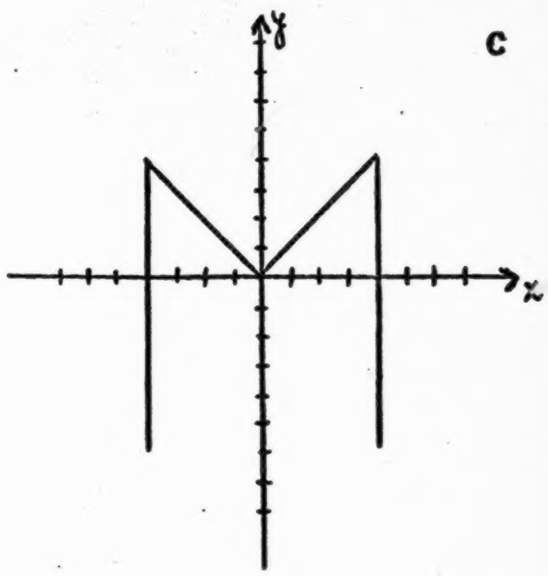
A. -10 B. 10 C. 9 D. -9 E. 11

22. To which one of the following expressions is $\frac{1}{ab}\sqrt{a^3b + ab^3}$ equivalent?

A. $\frac{a + b}{ab}\sqrt{ab}$ B. $2\sqrt{ab}$ C. $\sqrt{\frac{a}{b} + \frac{b}{a}}$ D. $\sqrt{a^2 + b^2}$ E. $a + b$

C

23. The mathematics class at Midland High School graphed an "M" as shown in the diagram at the right. A description of the "M" is the union of two graphs. One of these is the graph of the points (x,y) where $|x| = 4$ and $-6 \leq y \leq 4$. What is the other graph?



A. $x = y$ and $y = -x$, $-6 \leq y \leq 4$

B. $|x| = y$, $-4 \leq x \leq 4$

C. $|y| = x$, $-6 \leq y \leq 4$

D. $x = y$ or $-x = -y$

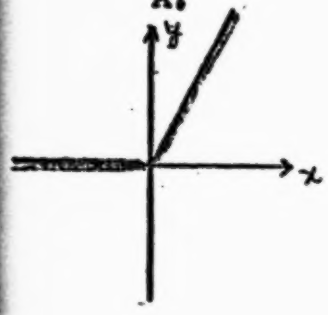
E. $y = x$, $-4 \leq x \leq 4$

24. If it takes John x days to do a particular task and it takes Carl y days to do it, how many days would it take them if they worked on the job together?

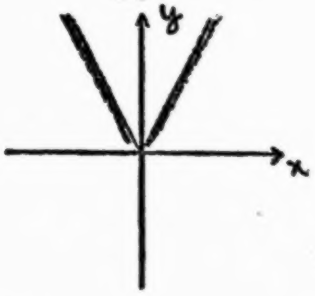
A. $\frac{x + y}{2}$ B. $\frac{1}{x + y}$ C. $\frac{1}{x} + \frac{1}{y}$ D. $\frac{1}{\frac{1}{x} + \frac{1}{y}}$ E. $x + y$

25. If $f(x) = x + |x|$, which one of the following represents the graph of $f(x)$?

A.



B.



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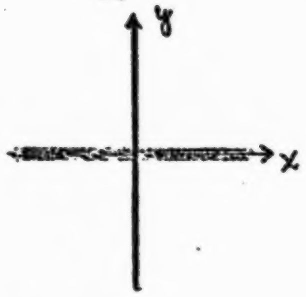
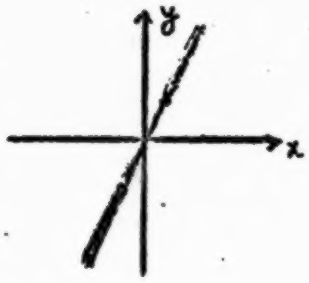
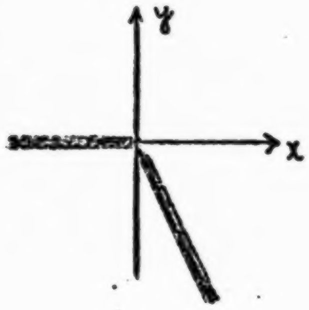


D.



E.



Code: XXXVIII-B-329

MINNESOTA NATIONAL LABORATORY

MATHEMATICS TEST

LEVEL TWO FORM D

(Grade Nine, 1964 Edition)

General Instructions:

This is a test of how well you understand and can apply what you have learned about mathematics. You are to read and try to answer each of these questions, even though some of the questions may be about things you have not yet studied.

Do not make any marks on the test booklet itself.

You are to indicate your answer only on the Answer Card using the special electrographic pencil you have been given.

Do your figuring only on the scratch paper you have been given.

First, print your name in the space provided on the Answer Card.

Now, read carefully the test procedures given below.

Test Procedures:

1. Read each item carefully and completely, think about the problem and do any figuring you want on the scratch paper.
2. Read the alternative answers carefully and select the one correct answer for each question.
3. Indicate your answer on the Answer Card by blackening the space over and around the letter (A, B, C, etc.) of the answer you have chosen, using the electrographic pencil. Be sure that the row in which you mark the answer has the same number as the question. If you wish to change an answer, be sure to erase the previous answer completely.
4. Since your score depends upon the number of correct answers that you mark, you should try to answer each question. However, do not spend too much time on any one item. If you have difficulty determining the correct answer, make the most careful guess you can and go on to the next item. If you finish before time is called, you can go back and work on the items that were more difficult. Try to do as well as you can.

If you have any questions about what you are to do, ask your teacher now.

Do not begin working until the teacher tells you to start.

1. The formula $F = \frac{9}{5}C + 32$ may be used to convert a temperature reading on a Centigrade thermometer to a temperature reading on a Fahrenheit thermometer. What is the temperature reading on a Centigrade thermometer when the reading on a Fahrenheit thermometer is $-15°$?

A. $-59°C$ B. $-107°C$ C. $9\frac{4}{9}°C$ D. $-84\frac{2}{5}°C$ E. $-26\frac{1}{9}°C$

2. The letters used below represent real numbers. Which one of the following mathematical sentences is true if A, B, and C are not equal to zero?

A. $\frac{A + B}{C} = \frac{A}{C} + \frac{B}{C}$ B. $\frac{1}{A} + \frac{1}{B} = \frac{1}{A + B}$ C. $\sqrt{A^2 + B^2} = A + B$

D. $A(B + C)D = AB + CD$ E. $A(BC) = (AB)(AC)$

3. Which one of the statements below is not always correct?

A. If $a = b$, then $ac = bc$.

B. If $a = b$, then $a + c = b + c$.

C. If $ac = bc$, then $a = b$.

D. If $ab = 0$, then either $a = 0$ or $b = 0$.

E. If $a = b$, then $a^2 = b^2$.

4. If $x + y = 6$ and $x > y$, what must be true of all x ?

A. $x \geq 6$ B. $x > 3$ C. $6 > x > 3$ D. $6 > x > 0$ E. No conclusion can be drawn.

5. If x is an even number, what is true of the expression $ax^2 + bx$?

A. It is even if either a or b is even, but not both.
B. It is odd if either a or b is odd, but not both.
C. It is always odd.
D. It is always even.
E. It is odd if either a or b is even, but not both.

6. Look at the three equations given below:

1. $(x + 3)^2 = x^2 + 6x + 9$

2. $x^2 - 3x = -2$

3. $3 + x^2 = x^2 - 2$

Which one of the statements below is true about these equations?

A. 1 is true for some values of x, 2 is true for every value of x, and 3 is never true.

B. 1 is always true, 2 is true for some values of x, and 3 is never true.

C. Each equation is true for every value of x.

D. Each equation has exactly two solutions.

E. 1 and 3 are true for some values of x, and 2 is never true.

7. One fifth of the sum of 12 and a certain number equals one fourth of the sum of 6 and twice the number. Which one of the following can be solved to find the number?

A. $\frac{12}{5} + n = \frac{6}{4} + 2n$

B. $\frac{12 + n}{5} = \frac{6 + 2n}{4}$

C. $\frac{12 + n}{5} = \frac{3 + n}{4}$

D. $\frac{n}{5} + 12 = \frac{2n}{4} + 6$

E. $5(12 + n) = 4(6 + 2n)$

8. Which one of the following systems is closed?

A. The set $\{0,1,2,3,\ldots\}$ where the operation is subtraction.

B. The set of perfect squares where the operation is addition.

C. The set $\{0\}$ where the operation is addition.

D. The set of odd integers where the operation is subtraction.

E. The set of positive even integers where the operation is division.

D

9. What is the sum of 7^{20} and 7^{21} ?

A. 7^{41} B. 14^{41} C. 49^{420} D. $8 \cdot 7^{20}$ E. $2 \cdot 7^{41}$

10. What is the complete solution of $x(x - 4) = 3(x - 4)$?

A. 3 B. 0 or 4 C. 3 or 4 D. 0 E. There are an infinite number of solutions.

11. If $x \neq 0$, which one of the following is equivalent to $\dfrac{\sqrt{x} + \sqrt{4x}}{\sqrt{x}}$?

A. 3 B. $\sqrt{5}$ C. $x + 2$ D. $\sqrt{4}$ E. $2\sqrt{x}$

12. Which one of the following is always true if a, b, and c are elements of {1,2,3,4,5,...} ?

A. If $a > b$, then $\frac{c}{a} < \frac{c}{b}$

B. If $a > b > c$, then $\frac{1}{a} + \frac{1}{b} > \frac{1}{b} + \frac{1}{c}$

C. If $a > b > c$, then $\dfrac{1}{(a + c)} < \dfrac{1}{(a + b)}$

D. If $a > b > c$, then $\frac{b}{c} > \frac{a}{c}$

E. If $a > b$, then $\frac{1}{a} > \frac{1}{b}$

13. What is the value of x in the following equation? $\frac{5}{6} - \frac{1}{2} - 8x > \frac{1}{3}$

A. $x = -\frac{1}{8}$ B. $x > \frac{1}{8}$ C. $x < 0$ D. $x > 0$ E. $x < -\frac{1}{8}$

14. Given two real numbers, let s represent the operation of squaring the first number and then adding this product to the second number.

Examples: $a \text{ s } b = a^2 + b$

$b \text{ s } a = b^2 + a$

Which one of the following statements is true?

A. The operation s is commutative only.

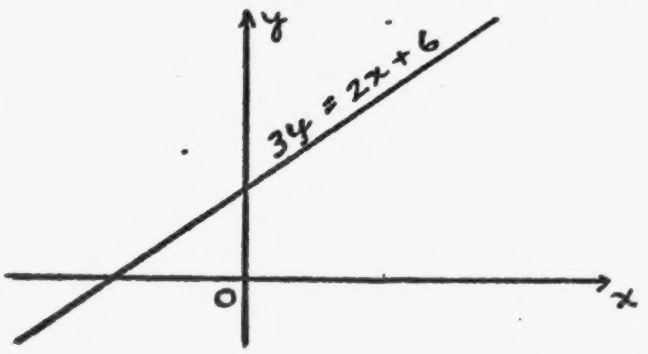
B. The operation s is associative only.

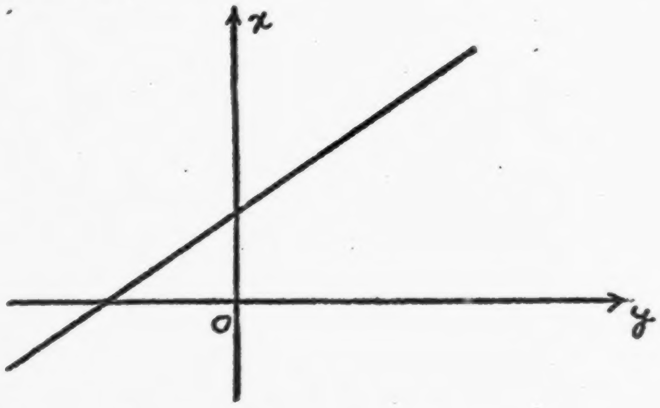
C. The operation s is both commutative and associative.

D. The system is not closed.

E. The operation s is not associative nor commutative.

15.





The diagram at the left above is the graph of the line $3y = 2x + 6$. In the diagram at the right above the x- and y- axes have been reversed, but the line still remains in the same position. What is the equation of the line with reference to the new axes?

A. $3y = 2x + 6$

B. $2y = 3x + 6$

C. $3x = -2y - 6$

D. $3x = -2y + 6$

E. $3x = 2y + 6$

16. The base of a triangle has the same length as the side of a square. A second side of the triangle is twice as long as the base, and the third side of the triangle is 12 inches shorter than three times the base. If the perimeter of the triangle equals that of the square, what is the perimeter of the square?

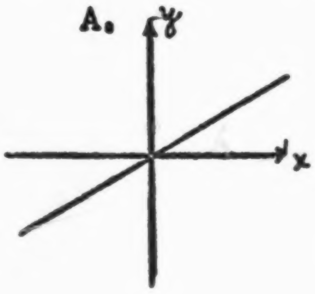
A. 24 inches B. 12 inches C. 36 inches D. 16 inches E. 20 inches

17. If $(x - 1)(x - k) = x^2 + x + k$, what is the value of k ?

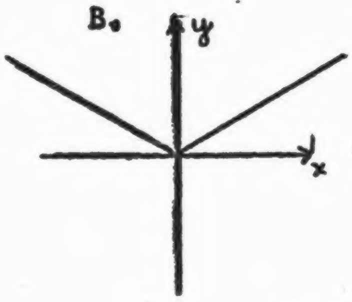
A. 1 B. 2 C. 3 D. -2 E. -1

18. Which one of the graphs below is the graph of $y = |\frac{x}{4}|$?

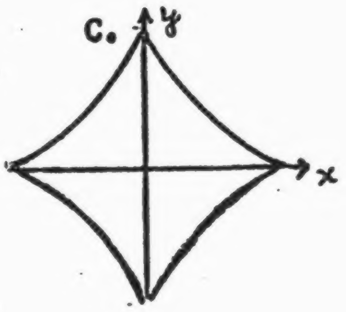
A.



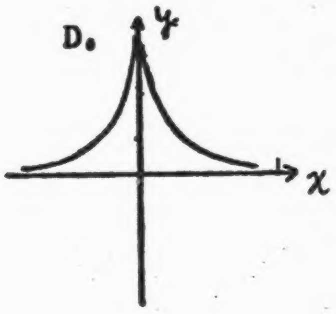
B.



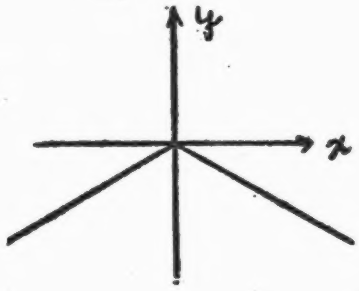
C.



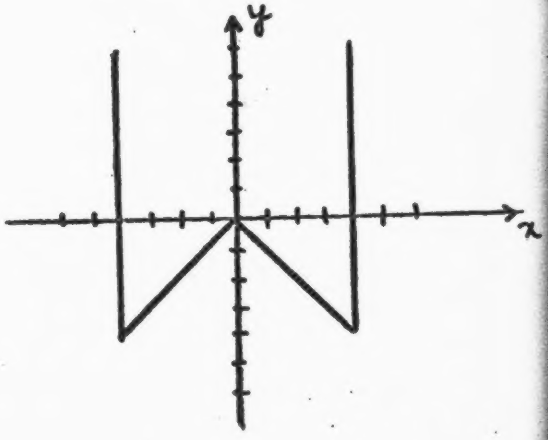
D.



E.



19. The mathematics class at Widland High School graphed a "W" as shown in the diagram at the right. A description of the "W" is the unicn of two graphs. One of these is the graph of the points (x,y) where $|x| = 4$ and $-4 \leq y \leq 6$. What is the other graph?



A. $|y| = x, \quad -4 \leq y \leq 6$

B. $y = x$ and $y = -x, \quad -4 \leq y \leq 6$

C. $|x| = y, \quad -4 \leq y \leq 6$

D. $x = y$ or $-x = -y$

E. $y = -|x|, \quad -4 \leq x \leq 4$

20. If

i) $f(x,y) = 1$ when $x > y$,
ii) $f(x,y) = 0$ when $x = y$,
iii) $f(x,y) = -1$ when $x < y$,

what is the value of $f(3,0) + f(3,3) + f(0,3)$?

A. 12 B. 9 C. 0 D. 3 E. 6

21. What must be true of all values of x which satisfy the inequality

$$(x + 2)(x - 3) < 0 \text{ ?}$$

A. $x > -2$ B. $x \geq 3$ or $x \leq -2$ C. $x < -2$ D. $-2 < x < 3$

E. $x < -2$ or $x < 3$

22. If $|x + y| = x$ and $y \neq 0$, what is the value of y?

A. $2x$ B. $-2x$ C. x D. $x - |x|$ E. $\pm x$

23. A boy can paint a fence in 5 hours and his brother can paint it in 3 hours. How long will it take to paint the fence if the brothers work together?

A. 4 hours B. 2 hours C. 8 hours D. $1\frac{7}{8}$ hours E. $1\frac{3}{5}$ hours

24. What values of x satisfy both of the following inequalities?

$$x^2 < 9 \quad \text{and} \quad x + 3 \leq 1$$

A. $-3 < x \leq -2$ B. $2 \leq x < 3$ C. $-3 < x \leq 2$ D. $-2 \leq x < 3$

E. The two solution sets have no elements in common.

25. Addition and multiplication for "remainder 3" arithmetic are defined by the following tables:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

x	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

A linear equation $ax + b = c$ can be solved in this system. For example, if $x + 2 = 1$, then $x = 2$. What is the solution of x in this system for the equation $2x + 1 = 2$?

A. 0 B. 3 C. 1 D. 0 or 1 E. 2

Code: XXXVIII-B-330

MINNESOTA NATIONAL LABORATORY

MATHEMATICS TEST

LEVEL FOUR FORM J

(Grade Eleven, 1964 Edition)

General Instructions:

This is a test of how well you understand and can apply what you have learned about mathematics. You are to read and try to answer each of these questions, even though some of the questions may be about things you have not yet studied.

Do not make any marks on the test booklet itself.

You are to indicate your answer only on the Answer Card using the special electrographic pencil you have been given.

Do your figuring only on the scratch paper you have been given.

First, print your name in the space provided on the Answer Card.

Now, read carefully the test procedures given below.

Test Procedures:

1. Read each item carefully and completely, think about the problem and do any figuring you want on the scratch paper.
2. Read the alternative answers carefully and select the one correct answer for each question.
3. Indicate your answer on the Answer Card by blackening the space over and around the letter (A, B, C, etc.) of the answer you have chosen, using the electrographic pencil. Be sure that the row in which you mark the answer has the same number as the question. If you wish to change an answer, be sure to erase the previous answer completely.
4. Since your score depends upon the number of correct answers that you mark, you should try to answer each question. However, do not spend too much time on any one item. If you have difficulty determining the correct answer, make the most careful guess you can and go on to the next item. If you finish before time is called, you can go back and work on the items that were more difficult. Try to do as well as you can.

If you have any questions about what you are to do, ask your teacher now.

Do not begin working until the teacher tells you to start.

1. What type of function is f(x) if the sum of the coordinates is 5 for each and every point that lies on the graph of f(x) ?

   A. A constant function.

   B. A logarithmic function.

   C. A quadratic function.

   D. A cubic function.

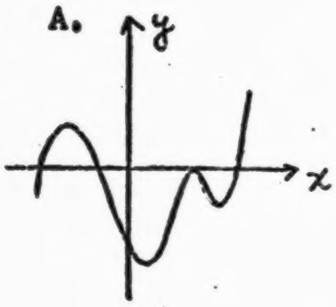
   E. A linear function.

2. Given that $\log_{10} 2 = .3010$ and $\log_{10} 3 = .4771$, what is the approximate value of $\log_{10} 12$?

   A. 1.8060 B. 1.9084 C. 1.0791 D. 1.1032 E. 1.2476

3. Which one of the following graphs is the most accurate representation of the graph of $f(x) = (x + 3)(x - 3)(x - 4)(x + 2)$?

A.



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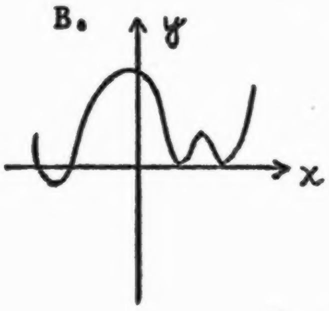
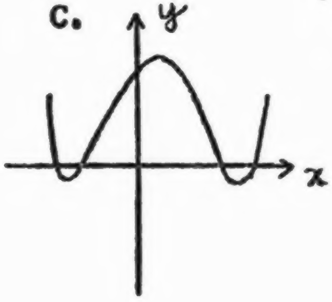


C.



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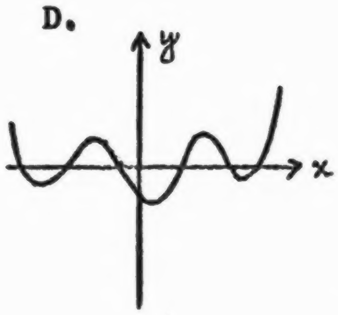
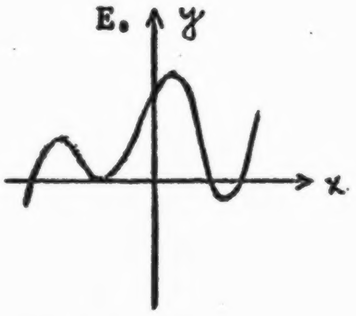
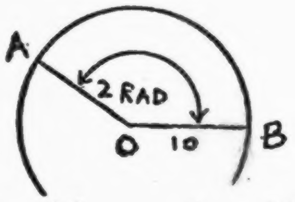


E.



4. 

Circle O has a radius of 10 inches. $\widehat{AB}$ subtends a central angle, AOB, whose measure is two radians. What is the measure in inches of $\widehat{AB}$?

   A. 20 B. 10π C. $\frac{20\pi}{3}$ D. $\frac{20}{\pi}$ E. 6π

5. Which one of the following expressions is equivalent to $2^{2x}(2^3 + 2^{x+3})$?

A. $2^{6x} + 2^{(2x^2+6x)}$

B. $4^{6x} + 4^{(2x^2+6x)}$

C. $2^{5x} + 2^{3x+3}$

D. 2^{2x^2+12x}

E. $2^{2x+3} + 2^{3x+3}$

6. What is the value of $(5)(16^{\frac{1}{6}})(2^{\frac{1}{3}})$?

A. $5\sqrt{32}$ B. $\sqrt{160}$ C. $5\sqrt[18]{32}$ D. 10 E. 20

7. For which values of x is it true that $\cos^2 x - \sin^2 x = \cos^2 x + \sin^2 x$?

A. No values of x

B. Only 0°

C. $n\frac{\pi}{2}$ where $n = 0, \pm 1, \pm 2, \ldots$

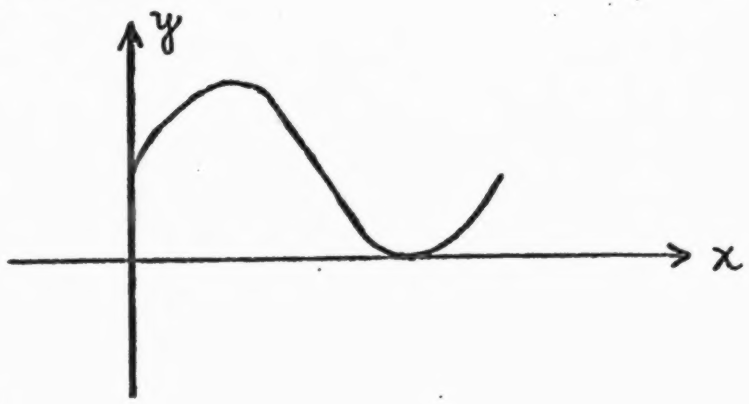
D. $n\pi$ where $n = 0, \pm 1, \pm 2, \ldots$

E. $\frac{3\pi}{2} + n \cdot 2\pi$ where $n = 0, \pm 1, \pm 2, \ldots$

8. Solve the following equation for x: $x \log 5 = \log 25$

A. log 20 B. log 5 C. 5 D. 2 E. The base must be known before the problem can be solved.

9. The graph on the right is a sketch of a period for which one of the following functions?



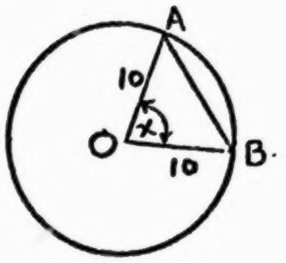
A. $y = 2 \sin x + \cos x + 1$

B. $y = 2 \sin x$

C. $y = -\cos x$

D. $y = \sin(x + 1)$

E. $y = 1 + \sin x$

10. 

The circle shown in the drawing has its center at O and a radius whose measure is 10 units. If the cosine law is used, what is the length of chord AB ?

A. 8 B. 10 C. $2\sqrt{10}$ D. $(2\sqrt{10})\cos x$ E. $10\sqrt{2 - 2\cos x}$

11. How should the fraction $\frac{1}{\sqrt{3} + \sqrt{2}}$ be written so that the denominator is rational?

A. $\sqrt{3} - \sqrt{2}$ B. $\sqrt{3} + \sqrt{2}$ C. $\frac{\sqrt{5}}{5}$ D. $\frac{\sqrt{6}}{6}$ E. $\sqrt{5}$

12. Consider the equation $2x^2 + 7x - 8 = kx$. For what value of k will the roots of this equation be of the same numerical value but opposite in sign?

A. 8 B. 7 C. $\frac{7}{2}$ D. -4 E. None of these

13. Consider the following quadratic equation: $(3^x)^2 - 12(3^x) + 27 = 0$
What are the values of x that make the equation true?

A. 9 and 3

B. $6 \pm 3\sqrt{7}$

C. 1 and 2

D. 1 and 1

E. None of the preceding is correct.

14. If $f(x) = 4x^2 + 4x + k$ and $f(\frac{1}{2}) = 0$, what is $f(k)$?

A. 21 B. 0 C. 36 D. $48\frac{1}{2}$ E. $4x^2 + 4x + \frac{1}{2}$

15. What is the solution set of $2x^2 + 7x - 4 < 0$?

A. $-4 < x < 1$

B. $-4 < x < \frac{1}{2}$

C. $x < -\frac{1}{2}$ or $x < 4$

D. $x < \frac{1}{2}$ or $x < -4$

E. $x > \frac{1}{2}$ or $x < -4$

16. Which one of the expressions given below is equivalent to $\frac{\sin A}{1 + \cos A} + \frac{1 - \cos A}{\sin A}$?

A. $\frac{2 \sin A}{1 + \cos A}$

B. -1

C. $\frac{1 - \cos A}{1 + \cos A}$

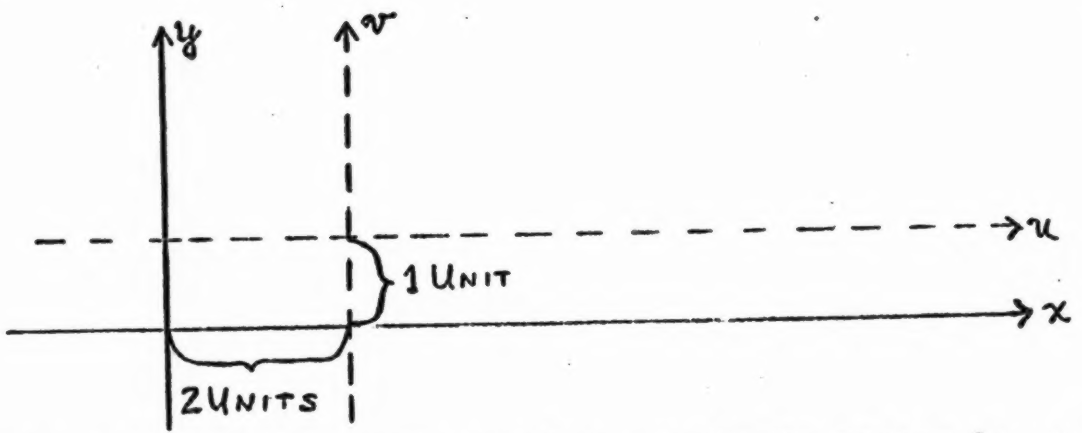
D. $\frac{\sin A + 1 - \cos A}{1 + \cos A + \sin A}$

E. $\frac{\sin A - 1}{1 - \cos A}$

17. A geometric progression has x as its first term and y as its second term. What is the fifth term of this progression?

A. $\frac{x^5}{y}$ B. $\frac{y^4}{x^3}$ C. $a + 4(y - x)$ D. $\frac{y^3}{x^4}$ E. $\frac{x^3}{y^4}$

18.



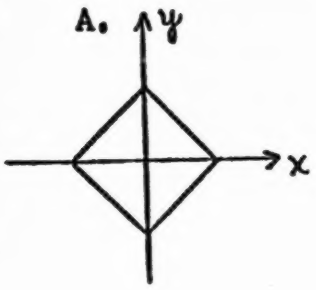
The equation of a line in the xy-rectangular coordinate system is $y = 2x + 4$. What is the equation of this line with reference to a uv-rectangular coordinate system, if the u-axis is parallel to the x-axis and 1 unit above it, and the v-axis is parallel to the y-axis and 2 units to the right of the y-axis?

A. $v = 2u + 4$ B. $v = 2u + 5$ C. $v = 2u + 6$ D. $v = 2u + 7$

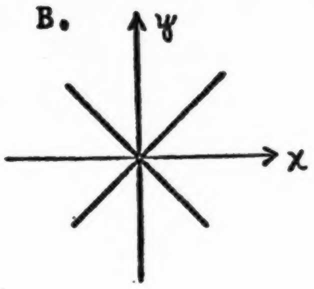
E. $v = 2u + 8$

19. Which one of the following graphs best describes the graph of $y = |x + 1|$?

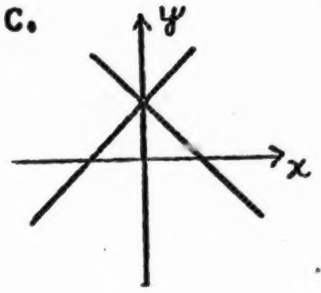
A.



B.



C.



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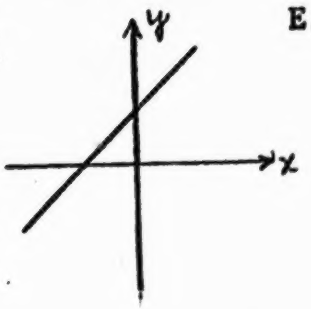
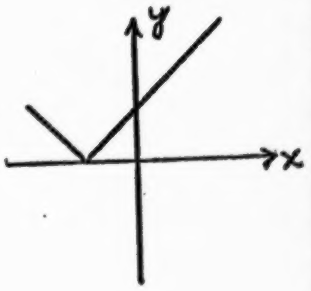


E.



20. If $f(x) = x + 1$ and $g(x) = x^2$, what is $g(f(x)) - f(g(x))$?

A. 0 B. $2x$ C. 1 D. $x^2 - (x + 1)$ E. $g\cdot(x + 1) - f\cdot x^2$

21. What is the smallest positive angle θ such that $0° < \theta \leq 360°$ when the following condition exists: $\cot \theta = -\sqrt{3}$?

A. 330° B. 240° C. 210° D. 150° E. 120°

22. Which one of the following is not an axiom for complex numbers? Assume x, y, and z represent complex numbers.

A. $x + y = y + x$

B. $xy = yx$

C. $(x + y) + z = x + (y + z)$

D. If $x < y$, and $y < z$, then $x < z$.

E. If $x = y$ and $y = z$, then $x = z$.

23. A periodic function has the following values: $f(0) = 2$, $f(\frac{1}{2}) = 2\frac{1}{2}$, $f(3) = -1$, $f(4) = 2\frac{1}{2}$, $f(6\frac{1}{2}) = -1$. What is the value of $f(-3)$?

A. 1 B. -1 C. 2 D. $2\frac{1}{2}$ E. 0

24. The geometric figure with a given perimeter P that encloses the greatest possible area is a circle of circumference P. If we consider only triangles, then the triangle with perimeter P that encloses the greatest possible area is the triangle whose sides are of equal length. If only figures of four sides are considered, then the figure with perimeter P that encloses the greatest possible area is a square.

On the basis of the information given above, which one of the following statements is true?

A. The area of a triangle is less than the area of a square.

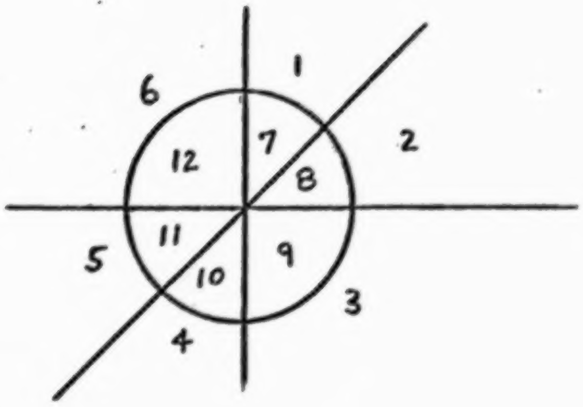
B. The area of a triangle is equal to that of a square if and only if their perimeters are equal.

C. The area of a triangle is less than the area of a circle if and only if the circle passes through all three vertices of the triangle.

D. The area of a square is less than the area of a circle only if their perimeters are equal.

E. If the perimeter of a circle is the same as that of a triangle, then the area of the triangle is less than the area of the circle.

25.



Note that the axes and graphs of the equations $x^2 + y^2 = 1$ and $x = y$ split the plane into twelve distinct regions, six of which are in the interior and six in the exterior of the circle. Which of these regions satisfy the inequalities

$$y > 0, \quad x^2 + y^2 > 1, \quad \text{and} \quad y > x \ ?$$

A. 1, 2, 6　　B. 1 only　　C. 1, 6　　D. 7, 12　　E. 2 only

Code: XXXVIII-B-331

MINNESOTA NATIONAL LABORATORY

MATHEMATICS TEST

LEVEL FOUR FORM K

(Grade Eleven, 1964 Edition)

General Instructions:

This is a test of how well you understand and can apply what you have learned about mathematics. You are to read and try to answer each of these questions, even though some of the questions may be about things you have not yet studied.

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2. Read the alternative answers carefully and select the one correct answer for each question.
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4. Since your score depends upon the number of correct answers that you mark, you should try to answer each question. However, do not spend too much time on any one item. If you have difficulty determining the correct answer, make the most careful guess you can and go on to the next item. If you finish before time is called, you can go back and work on the items that were more difficult. Try to do as well as you can.

If you have any questions about what you are to do, ask your teacher now.

Do not begin working until the teacher tells you to start.

K

1. Which one of the following is the expression for $\sin 879°$ when it is expressed in terms of a function of a first quadrant angle?

A. $\sin 21°$ B. $\cos 21°$ C. $\sin 69°$ D. $-\sin 69°$ E. $-\sin 21°$

2. If p and q are the roots of the equation $x^2 + bx + c = 0$, then what is the value of $p + q$?

A. b B. $-b$ C. $\frac{b}{b + c}$ D. c E. $b^2 - 4ac$

3. If $|x - 1| = 2$, what number (or numbers) does x represent?

A. 3 only B. -1 only C. -3 only D. -1 or 3 E. Cannot be determined.

4. Given the graph of the equation $y = x^2 - 2x - 3$, as shown in the drawing, which values of x satisfy the inequality $x^2 - 2x - 3 \geq 0$?

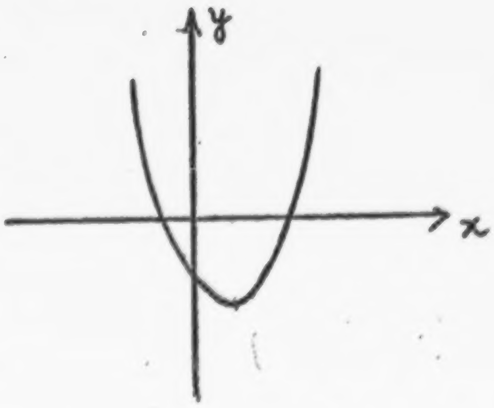
A. $x \geq 3$

B. $-1 \leq x \leq 3$

C. $x \geq -1$ or $x \geq 3$

D. $x \geq 3$ or $x \leq -1$

E. $x \leq -1$ or $x \leq 3$



5. Which one of the following is equivalent to $\frac{\log (a \cdot b)}{\log a + \log b}$?

A. 1 B. 2 C. $\frac{ab}{a + b}$ D. $\log (a \cdot b) - \log a - \log b$ E. $\frac{\log (a \cdot b)}{a \cdot b}$

6. If $\log_y ab = 2$, what is the value of y?

A. $\frac{ab}{2}$ B. 10 C. ab^2 D. $2ab$ E. $\sqrt{ab}$

7. If $f(x) = 7\pi \sin(2x + \pi)$, what is the period of $f(x)$?

A. 2 B. $-\frac{\pi}{2}$ C. 7π D. 4π E. π

8. What values of t make the expression $(t^2 - 1)^2x^2 + (t - 1)x + t + 1$ a quadratic function?

A. All real values of t.

B. All real values of t except $t = 1$.

C. All real values of t except $t = -1$.

D. All real values of t except $t = 1$ and $t = -1$.

E. All real values of t except $-1 < t < 1$.

9. If the symbol $\sin^{-1}x$ is used to represent the inverse sine of x, what is the value of $\cos(\sin^{-1} -.5)$ if we restrict the range of $\sin^{-1}x$ to $\frac{\pi}{2} \geq x \geq -\frac{\pi}{2}$?

A. $-\frac{\sqrt{3}}{2}$ B. $\frac{\sqrt{3}}{2}$ C. $\frac{\pi}{6}$ D. $-\frac{\pi}{6}$ E. $\cot^{-1}(-.5)$

10. If $x^2 > y^2$ and $y > 0$, which one of the following inequalities expresses a relationship between x and y that is always true?

A. $x > y$ B. $x > \pm y$ C. $x > \frac{y^2}{x}$ D. $x > y$ or $x < -y$

E. $x < y$ or $x > -y$

11. Which one of the following equations, when graphed for one period, will have an appearance as shown in the sketch at the right?

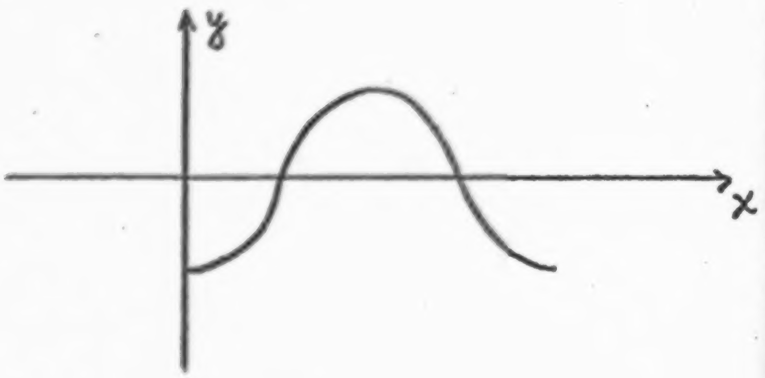
A. $y = \sin(-x)$

B. $y = \cos(-x)$

C. $y = -\cos x$

D. $y = -1 + \sin x$

E. $y = \sin x + \cos x$



12. Suppose a coordinate system is constructed as follows: We let our reference system consist of one line segment with end points A and B and a length of 10 units. Let point P be any point in the plane. Let x be the distance that P is from A and y be the distance that P is from B. Thus the location of point P is given by specifying the ordered pair (x, y). In this coordinate system what is the graph of the equation $y = 3$?

A. A single point

B. Two points

C. One line parallel to segment AB

D. Two parallel lines

E. A circle

13. Which one of the following pairs of inequalities has a graphical solution as indicated by the shaded region of the graph at the right?

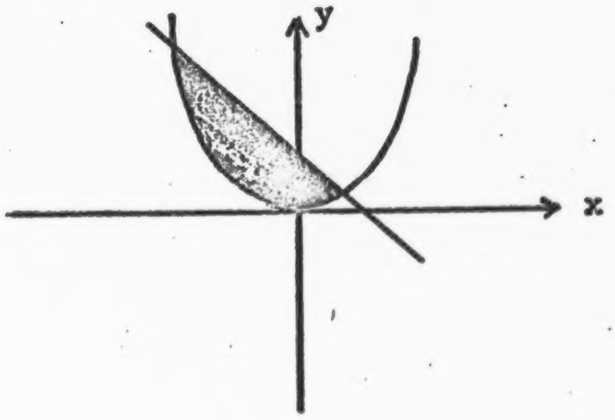
A. $x^2 \geq y$ and $y \leq x - 3$

B. $x^2 \geq y$ and $y \geq 3$

C. $y \geq x^2$ and $y \leq 2x - 6$

D. $y \geq x^2$ and $y \leq 3 - x$

E. $x^2 \geq y$ and $y \leq 6 - 2x$



14. If we know two angles and one side as shown in the diagram at the right, what is x ?

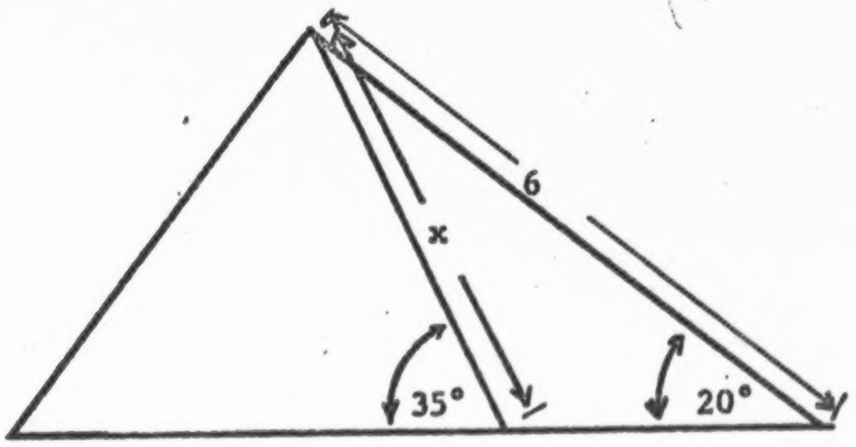
A. $6 \sin 20°$

B. $\dfrac{6 \sin 20°}{\sin 35°}$

C. $6 \sin 35°$

D. $\dfrac{6 \sin 35°}{\sin 20°}$

E. $\dfrac{\sin 20°}{6 \sin 35°}$

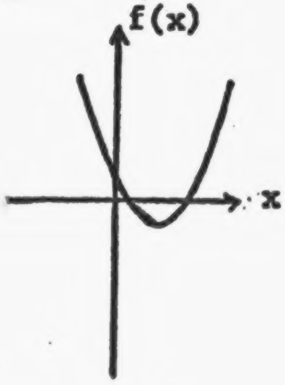


15. The expression $\dfrac{\cos(x + y) + \cos(x - y)}{\cos x \cos y}$ reduces to which one of the following?

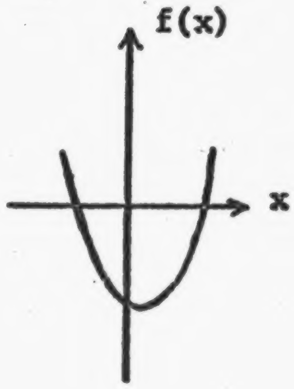
A. 2 B. $\dfrac{1}{\sin y}$ C. $\dfrac{1}{\cos y}$ D. $\dfrac{2}{y}$ E. $\dfrac{2}{\cos x}$

16. If $f(x) = x^2 + tx + t - 5$ and $f(3) = 0$, which one of the following is the best sketch of the graph $f(x)$?

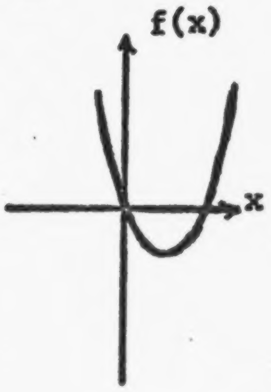
A.



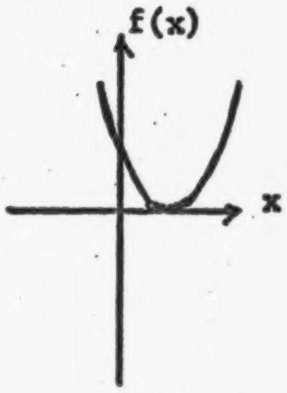
B.



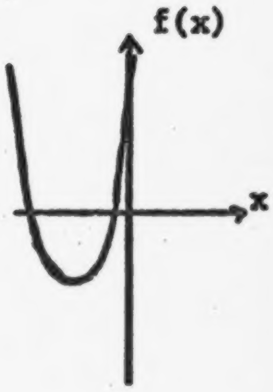
C.



D.



E.



17. What is the result if $\sqrt[3]{\frac{a}{b}}$ is divided by $\sqrt[4]{\frac{a}{b}}$ and the quotient is rationalized?

A. $\dfrac{\sqrt[4]{a^3b}}{b}$ B. $\sqrt{ab}$ C. $\dfrac{\sqrt[12]{ab^{11}}}{b}$ D. $\dfrac{\sqrt[12]{ab}}{b}$ E. $\dfrac{\sqrt[7]{b^3a^4}}{ab}$

18. Consider the following quadratic equation:

$$(3^x)^2 - 12(3^x) + 27 = 0$$

What are the values of 3^x that make the equation true?

A. 1 and 2

B. 9 and 3

C. $6 \pm 3\sqrt{7}$

D. 3 and 3

E. None of the preceding is correct.

19. How many values of x satisfy the equation $\cos 2x = 0$ if $0 \le x < 360°$?

A. 4 B. 3 C. 2 D. 1 E. 0

20. The volume of a sphere varies as the cube of its radius. If three solid metal spheres of radii a, b, and c respectively are combined to form a larger solid sphere, what is the radius of this sphere?

A. $(a + b + c)^3$

B. $\left(\frac{a + b + c}{3}\right)^3$

C. $\frac{a + b + c}{3}$

D. $\sqrt[3]{a + b + c}$

E. $\sqrt[3]{a^3 + b^3 + c^3}$

21. How do you write the expression $2^{3x} + 2^{(x^2 + 5x)}$ if 2^x is removed as a common factor?

A. $2^x(2^3 + 2^{x+5})$

B. $2^x(2^3 + 2^{x^2+5})$

C. $2^x(2^{2x} + 2^{x^2+4x})$

D. $2^x(1^3 + 1^{x+5})$

E. $2^x(2^{2x} + 2^{x+5})$

22. What is x if the numbers 1, x, and -1 form a geometric progression?

A. ± 1 B. -1 C. 0 D. $\pm \frac{1}{2}$ E. $\pm i$

23. If $(x + 5i) - (2 - 7i) = 2 + yi$, what is $x + y$?

A. 2 B. 8 C. 16 D. 32 E. $12i - 4 + y^2$

24. At a cafeteria counter a man has to choose between two items of food. The table below lists the four possible decisions he might make:

	take	not	take	not	take	not	take	not
Food A	x			x	x			x
Food B	x			x		x	x	
	Case I		Case II		Case III		Case IV	

If he goes to a third item, he may either take it or not. This doubles the total number of decisions, since each of the decisions in the table may be paired with taking the third item (4 possibilities) or not (4 possibilities). How many possible decisions might he make if N items of food are offered and he takes at least one item?

A. $2^N - 1$ B. 2^{N-1} C. 2^N D. 3^N E. Not enough information is give

25. Dividing a number by $3 + \sqrt{2}$ gives the same results as if the number were multiplied by which one of the following?

A. $\frac{3}{7} - \frac{\sqrt{2}}{7}$

B. $-3 - \sqrt{2}$

C. $-\frac{1}{3} - \frac{1}{\sqrt{2}}$

D. $\frac{1}{3} + \frac{1}{\sqrt{2}}$

E. $3 - \sqrt{2}$

Code: XXXVIII-B-332

MINNESOTA NATIONAL LABORATORY

MATHEMATICS TEST

LEVEL FOUR FORM L

(Grade Eleven, 1964 Edition)

General Instructions:

This is a test of how well you understand and can apply what you have learned about mathematics. You are to read and try to answer each of these questions, even though some of the questions may be about things you have not yet studied.

Do not make any marks on the test booklet itself.

You are to indicate your answer only on the Answer Card using the special electrographic pencil you have been given.

Do your figuring only on the scratch paper you have been given.

First, print your name in the space provided on the Answer Card.

Now, read carefully the test procedures given below.

Test Procedures:

1. Read each item carefully and completely, think about the problem and do any figuring you want on the scratch paper.
2. Read the alternative answers carefully and select the one correct answer for each question.
3. Indicate your answer on the Answer Card by blackening the space over and around the letter (A, B, C, etc.) of the answer you have chosen, using the electrographic pencil. Be sure that the row in which you mark the answer has the same number as the question. If you wish to change an answer, be sure to erase the previous answer completely.
4. Since your score depends upon the number of correct answers that you mark, you should try to answer each question. However, do not spend too much time on any one item. If you have difficulty determining the correct answer, make the most careful guess you can and go on to the next item. If you finish before time is called, you can go back and work on the items that were more difficult. Try to do as well as you can.

If you have any questions about what you are to do, ask your teacher now.

Do not begin working until the teacher tells you to start.

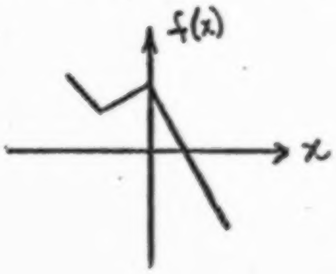
1. A line passes through the point (2,1) and has a slope of 3. What is the ordinate of a point on this line if its abscissa is 4 ?

A. 7 B. 6 C. 5 D. 4 E. 3

2. What is the product of $\sqrt{(2)(-3)}$ and $\sqrt{(2)(-5)}$?

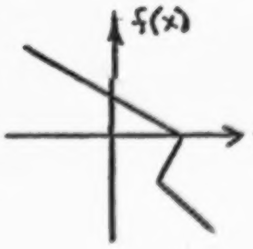
A. $\pm 2\sqrt{15}$ B. $2\sqrt{15}$ C. $2i\sqrt{15}$ D. $-2i\sqrt{15}$ E. $-2\sqrt{15}$

3. Given the graph of a function f(x) :

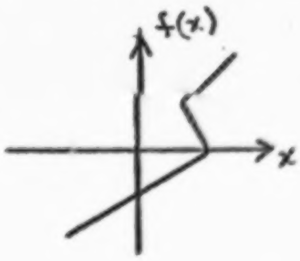


Which one of the following graphs is the graph of -f(x)?

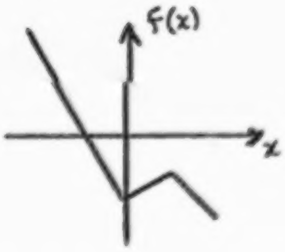
A.



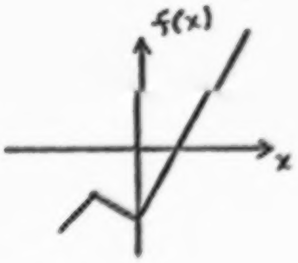
B.



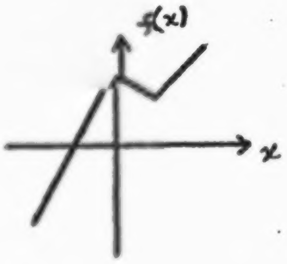
C.



D.



E.



4. What are the factors of $(a + 3)^2 - (b - 4)^2$?

A. a + 3 and b - 4

B. a + b - 1 and a + b - 1

C. a + b + 1 and a - b - 7

D. a + b - 1 and a - b - 1

E. a + b - 1 and a - b + 7

5. What is the sum of the roots of the equation $x(x + 1) = 12$?

A. 1 B. -1 C. 7 D. 11 E. 23

6. If $x < 1$, what is a simplification of the expression $|2x - 3| + |3 - 2x|$?

A. 0 B. 4x - 6 C. 6 - 4x D. 6 E. 4x

7. The following table gives numbers from 2.0 to 8.0 and their logarithms to the base 10 correct to two decimal places. What is the value of $10^{-.6}$ according to the table?

No.	Log	No.	Log	No.	Log
2.5	.40	4.5	.65	6.5	.81
3.0	.48	5.0	.70	7.0	.85
3.5	.54	5.5	.74	7.5	.88
4.0	.60	6.0	.78	8.0	.90

A. .25 B. -4.0 C. .078 D. -.078 E. .040

8. Which one of the following expressions is equivalent to $\frac{\sin(90° - w)}{\cos(90° - w)}$?

A. tan w B. sin w C. cos w D. cot(90° - w) E. cot w

9. What is the product $(x^{-1} + y^{-1})(x + y)^{-1}$ where $x \neq 0$, $y \neq 0$, and $x \neq y$?

A. $\frac{1}{xy}$ B. $\frac{1}{(x + y)^2}$ C. 1 D. $\frac{1}{x + y}$ E. $\frac{1}{x^2} + \frac{1}{y^2}$

10. What must be true of k if the equation $7x^2 + kx + k - 3 = 0$ has 0 as one root?

A. $k = 7$ B. $k < 1$ C. $k = 3$ D. $k^2 - 28k + 84 = 0$ E. $k = 0$

11. How many terms are there in the product $(a + b)^7 \cdot (a + b)^8$?

A. 15 B. 16 C. 17 D. 57 E. 9

12. If $(a,b) \Delta (c,d) = (a - c, b + d)$ and $(3,2) \Delta (0,0) = (x,y) \Delta (3,2)$, what is (x,y) ?

A. (0,0) B. (-3,-2) C. (3,2) D. (6,0) E. (2,3)

13. At how many points does the graph of $y = x^2 - 5$ intersect the graph of $x^2 + y^2 = 25$?

A. 4 B. 3 C. 2 D. 1 E. 0

14. We have 7 uniforms with different numbers on them to give to give to 4 football players. In how many different ways can we distribute them?

A. 4 B. 961 C. 2401 D. 28 E. 840

15. The equation $y = x^2 - 4x + 3$ defines a function. Let x be any element of the domain and y be the corresponding element of the range. If the domain is the set of all real numbers, what is the range of the function?

A. $y > 0$ B. $y < 0$ C. $y > -1$ D. $y > 3$ E. All real numbers

16. Let a, b, and c be the three sides of a triangle with $a = 2$, $b = 3$, and $c = 4$. Let angle A be the angle opposite side a. What is cos A ?

A. $\frac{3}{4}$ B. $\frac{2}{3}$ C. $\frac{1}{2}$ D. $\frac{7}{8}$ E. $\frac{4}{3}$

17. If $0 \leq x < 360°$, how many values of x satisfy the equation $\sin^2 x = \sin x$?

A. 3 B. 1 C. 2 D. 5 E. 0

18. What is the solution of $\frac{2}{x} < 3$?

A. $x > \frac{3}{2}$ B. $x > 0$ or $x > \frac{2}{3}$ C. $x > \frac{2}{3}$ D. $x > \frac{2}{3}$ or $x < 0$

E. $0 < x < \frac{2}{3}$

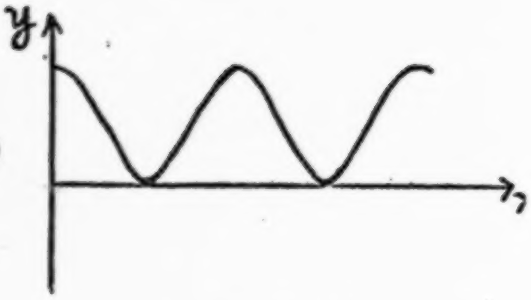
19. If $f(x) = x + 7$, and $g(x)$ is the inverse of $f(x)$, what is $g(3)$?

A. $\frac{1}{10}$ B. -10 C. -4 D. $-\frac{1}{10}$ E. 4

20. What must be true of k if the graph of the function $3x^2 + 2x + k$ intersects the x-axis ?

A. $k = 0$ B. $k \geq 0$ C. $k \leq \frac{1}{3}$ D. $k \geq 1$ E. $k = -\frac{1}{3}$

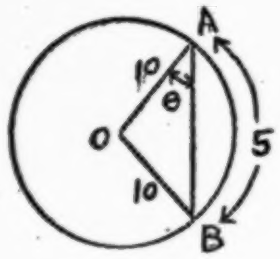
21. Which one of the following equations has a graph as shown in the sketch at the right?



A. $y = \sin^2 x$ B. $y = \cos^2 x$ C. $y = \sin(90° - x)$

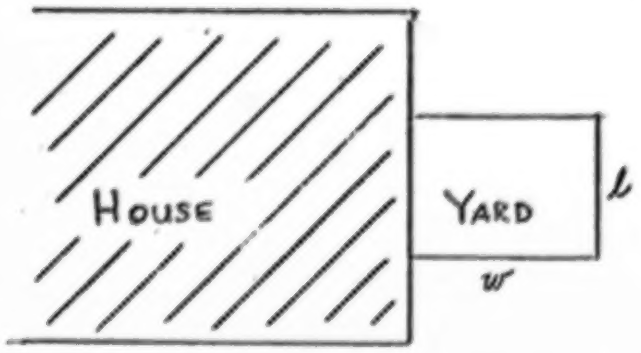
D. $y = \sin \frac{x}{2}$ E. $y = \cos \frac{x}{2}$

22. In circle O, $\overset{\frown}{AB}$ is 5 units in length. The radius of this circle is 10 units in length. What is the measure of angle θ ?



A. $\frac{2\pi - 1}{4}$ B. $\frac{5\pi}{12}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{6}$ E. $\frac{5\pi + 3}{10}$

23. Suppose we have 60 feet of fencing available to enclose a rectangular yard that will have a house serving as one side of the yard. We notice (see table below) that the area enclosed will depend upon how this 60 feet of fencing is used.



Length	Width	Area
50 ft.	5 ft.	250 sq. ft.
40 ft.	10 ft.	400 sq. ft.
30 ft.	15 ft.	450 sq. ft.

Which one of the following conclusions is correct?

A. The maximum area will be obtained if the yard is square.

B. The area of a rectangle is greater than the area of a square.

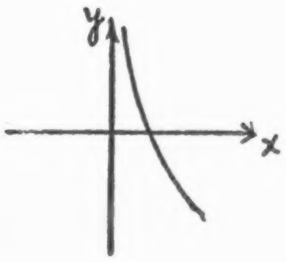
C. The area of the yard continues to increase as its width increases.

D. The maximum area will be obtained if the length is twice the width of the yard.

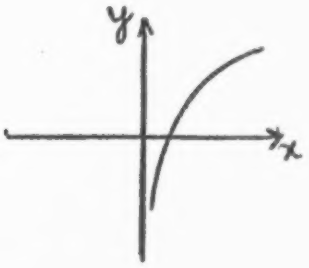
E. The area can be made as large as we choose.

24. Which one of the following is a sketch of the graph of $y = a^{\log_a x}$ where $a > 0$ but $a \neq 1$?

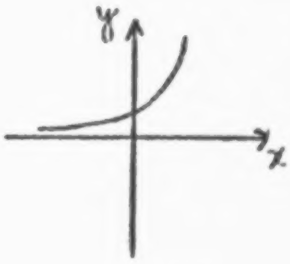
A.



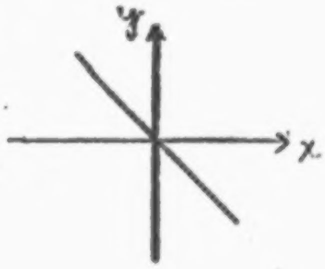
B.



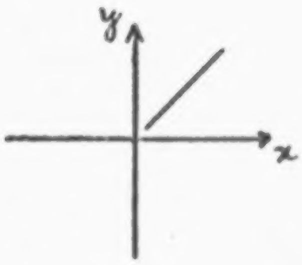
C.



D.



E.



25. By definition, $i^2 = -1$. Consider the following three statements.

1. If $i > 0$, then $i^2 > 0$ since the square of a positive number is positive. Since $i^2 = -1$, we have $-1 > 0$.
2. If $i < 0$, then $i^2 > 0$ again since the square of a negative number is positive and, since $i^2 = -1$, we have $-1 > 0$ again.
3. $i \neq 0$

As a result of the above statements and definition, which one of the following statements can definitely be stated as a theorem?

A. All complex numbers are positive.

B. The complex numbers can not be ordered.

C. $-1 > 0$ in the complex number field.

D. $i = \sqrt{-1}$ is the square root of a positive number.

E. A number of the form $a + bi$, where a and b are real, is positive or negative depending upon whether a and b are positive or negative.

Minnesota National Laboratory Mathematics Tests

(1965 Edition)

During the week of May 9 - May 13 you shall be receiving the materials for spring testing of pupils in classes participating in the Secondary Mathematics Evaluation Project (the 5 State Project). These tests should be administered as soon as possible after you receive them.

General Test Procedures (please read carefully):

This spring the tests to be administered are for the most part different from those previously used in this project. We believe these new tests will provide a much better picture of the specific program differences than the STEP.

Two different Coop mathematics achievement tests and one MNL (Minnesota National Laboratory) test are to be administered. Each of these separate tests wil require one class period. Teachers teaching two classes at the same grade level will receive only one set of each of the different tests which can be used for both classes.

The two Coop tests that you will be receiving are listed below according to grade level.

Specific instructions for administering the Coop tests and the MNL test are given below and further instructions will be included with the test materials. The same instructions apply to both Coop tests. However, the instructions and procedures for the MNL test will be somewhat different.

Please read over the following instructions and those included with the test materials carefully before administering each of the tests. If you have any questions, please telephone us collect.

Order of Administration:

The three tests are to be administered in a definite order. The two Coop tests should be administered before the MNL test. The Coop tests should be administered in the order in which they appear for each grade in the list below. For example, at the ninth grade, Structure of the Number System should be administered first, then Algebra I, and then the MNL test. In the 7th and 8th grades, the Structure of the Number System should be administered after Arithmetic, and the MNL test should be administered after Structure of the Number System. Please follo the order for administration of the two Coop tests and the MNL test appropriate for your grade level.

Answer Cards Identification:

You will be receiving, as before, mark-sense answer cards on which pupils are to record their answers. Since the name of the test is not on the answer cards, the answer cards have been color coded. It is very important that the correct answer cards be used with each test. For the Coop tests, each answer card has a single colored stripe along one border. The colored stripe that goes with each Coop test is indicated in the list of tests below. The MNL answer cards have two broad colored stripes (red & blue) in the middle of each card. Please use the correct color answer cards with each test.

Each pupil will only need one answer card for each test.

Grade	Test	Answer card (Border Color)
7 & 8	Arithmetic	YELLOW
	Structure of the Number System	BROWN
	MNL Test	(Special MNL Answer Card)
9	Structure of the Number System	PINK
	Algebra I	BLUE
	MNL Test	(Special MNL Answer Card)
10	Geometry I	GRAY
	Geometry II	GREEN
	MNL Test	(Special MNL Answer Card)
11	Algebra II	PURPLE
	Algebra III	ORANGE
	MNL Test	(Special MNL Answer Card)

For all of these tests, we would like you to encourage the pupils to do the very best they can. An adequate assessment of the different secondary mathematics programs depends upon pupils' performing as well as they can on these tests.

Test Instructions for the Coop and MNL Tests:

Coop Tests -

Specific instructions for administering these tests will be included wit. the test materials. These instructions should be followed exactly as given with the following exceptions:

1. Instead of answer sheets, answer cards are to be used.
2. Pupils are to put only their names on the answer cards.

Note: Pupils are to be allowed exactly 40 minutes working time on each Coop test.

MNL Tests -

Specific instructions for the MNL tests are on the test forms themselves. Particularly note the following:

1. The MNL tests should be distributed with the answer cards at the very beginning of the class period.

2. Pupils should fill in all information requested at the top of the MNL test answer card (name, sex and birthdate).

3. The instructions on the test forms should be read aloud at the same time that the pupils read the instructions to themselves.

4. Pupils should be given the full remainder of the class period to work the MNL tests.

5. For each MNL test, pupils are to make two responses to each question - a first and a second response. The first and the second responses are to be indicated in different places on the answer cards. The first response is to be the answer a pupil thinks is the most nearly correct alternative. The second response is to be the answer the pupil thinks is the next most nearly correct alternative.

In other words, a pupil's first response represents the answer that he would choose if only one response were required. The second response represents the pupil's next best choice among the remaining alternatives.

Pupils are always to make both responses, i.e. make a second choice even though they feel their first choice was the only correct choice. If a pupil does not make a second choice, his first choice will not count.

There is, however, only one correct answer to each question.

6. Any questions pupils have as to test procedures should be answered on an individual pupil basis.

Packaging & Returning Materials:

Please return all copies of each of the tests immediately after you have finished administering them. Those teachers who are teaching both a control and an experimental class should take care that the test materials of their two classes do not get mixed up in any way. Note: The answer cards will be in individual envelopes, properly identified for each test. Please return the answer cards in the designated envelopes. You will find return postage enclosed with the materials for your convenience.

Please write the following below the stamp:

SPECIAL FOURTH CLASS MAIL
OBJECTIVE TEST MATERIAL

MINNESOTA NATIONAL LABORATORY

MATHEMATICS TEST

FORM C

(Grade Nine, 1965 Edition)

Read these instructions carefully.

Print your full name (last name first), grade (7, 8, 9, 10, 11, or 12), and sex (M or F) in the spaces provided on your IBM answer card.

Look at your IBM answer card. Note that there are two columns for your answers. One column is labeled 1st choice, and the other column is labeled 2nd choice. When you work a problem, you may obtain two answers, one which you think is most likely correct and another which you think is next most likely correct. If so, you are to mark the first answer in the column labeled 1st choice and the second answer in the column labeled 2nd choice. Even if you do not have a second choice, guess one and mark your guess in the colum labeled 2nd choice. If you do not have a first or a second choice, guess them both and mark one guess in the column labeled 1st choice and the other guess in the column labeled 2nd choice. In other words, make sure that you mark two answers in the appropriate columns to every question. Using the electrographic pencil given you, make your marks heavily and so that they fill the entire spaces over and around the letters corresponding to your answers.

Do not write in the test booklet. Use scratch paper to do your figuring. You will have the remainder of the class hour to complete the test. If you have any questions, raise your hand; your teacher will come to you as soon as possible. Now begin.

C

1. The sum $a + b$ is smaller than a and also smaller than b. Which one of the following statements is correct?

A. Both a and b are positive.

B. Both a and b are negative.

C. a is positive and b is negative.

D. a is negative and b is positive.

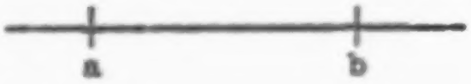
E. One of the numbers is zero.

2. Which one of the following is equivalent to the expression $4(k + 4)3$?

A. $4k + 7$ B. $4k + 12$ C. $7k + 28$ D. $4(k + 4) \quad (k + 4)3$

E. $12k + 48$

3. Suppose that a and b are the coordinates, respectively, of two points on a number line (see diagram at right).



Which one of the following is an expression for the coordinate of the point midway between them?

A. $\sqrt{b - a}$ B. $\frac{a + b}{2}$ C. $\frac{b + 2a}{2}$ D. $\sqrt{a^2 + b^2}$ E. $\frac{b - a}{2}$

4. Suppose $x(x - 4) = -3$. For which pair of values of x below will substitution of either value make this equation true?

A. $x = -3$ or $x = 1$ B. $x = 0$ or $x = 4$ C. $x = -3$ or $x = 4$

D. $x = 0$ or $x = 1$ E. $x = 3$ or $x = 1$

5. If y is an odd number, what is true of the expression $y^2 + ay$?

A. It is odd if a is even.

B. It is even if a is even.

C. It is odd if a is odd.

D. It is always even.

E. It is always odd.

6. Suppose $F = \frac{a}{b} + \frac{c}{d}$ and $d = kb$. Which one of the expressions below is equivalent to F?

A. $\frac{a + kc}{d}$ B. $\frac{k(a + c)}{d}$ C. $\frac{ka + c}{b}$ D. $\frac{a + kc}{b}$ E. $\frac{ka + c}{kb}$

7. The larger of two numbers is 17 more than twice the smaller number. When the larger number is divided by the smaller number, the quotient is 3, with a remainder of 7. If x is the smaller number, which one of the following equations should be used to determine x?

A. $2x - 17 = 10x$ B. $2x + 17 = 3x + 7$ C. $\frac{2x + 17}{3x} = 7$

D. $2x + 17 = 3x^2 + 7x$ E. $2x - 17 = 10$

8. What value or values of b will make $x = 1$ a solution of the following equation?

$$x^2 + 2b^2x - 5b + 1 = 0$$

A. $\frac{1}{2}$ B. 0 C. $\frac{1}{2}$ or 2 D. 0 or 2 E. 2

9. Definition: $|-a| = |a| = a$ if $a > 0$ and $|-a| = |a| = -a$ if $a < 0$. Thus, the equation $|x-y| = x$ implies not only that $x > 0$ but also that, if $y \neq 0$, y is equal to

A. $2x$ B. x C. $-2x$ D. $x - |x|$ E. $\pm x$

10. What is the smallest integer x which satisfies the following inequality? (Integers are positive or negative whole numbers.)

$$\frac{5}{7} > \frac{x}{-14}$$

A. -10 B. 10 C. 9 D. -9 E. 11

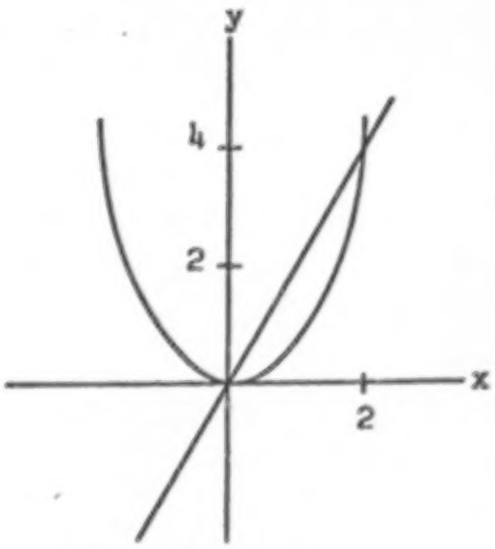
11. A portion of the graph at the right is a graph of the following inequality:

$$x^2 < 2x$$

For what values of x is this inequality true?

A. $x > 0$ B. $x < 0$ C. $x < 0$ and $x > 2$

D. $x < 2$ E. $0 < x < 2$



12. Let L be the least common multiple of 3 and 5 and G be the greatest common divisor of 3 and 12. What is the greatest common divisor of L and G?

A. 3 B. 5 C. 12 D. 15 E. 36

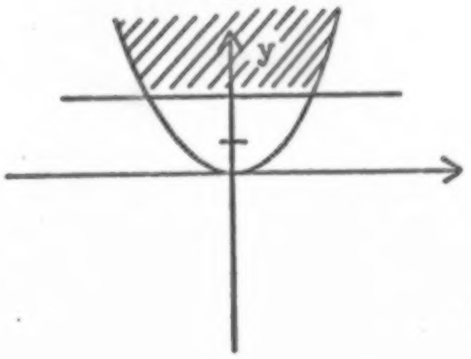
13. The area A of a triangle is equal to one half the product of its altitude a and its base b; that is, $A = \frac{1}{2}ab$. If $A = 48$ and $a \leq 12$, which one of the following statements is true?

A. $b \geq 4$ B. $4 < b < 8$ C. $b \geq 8$ D. $b \leq 4$ E. $b \leq 8$

14. Which one of the following is a graph of the intersection of the following system of inequalities? (Intersections are indicated by shaded areas.)

$$y \geq x^2$$
$$y \leq 2$$

A.



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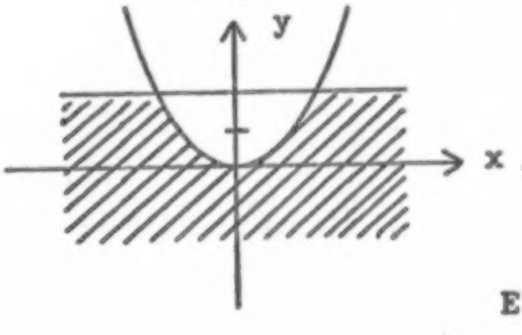

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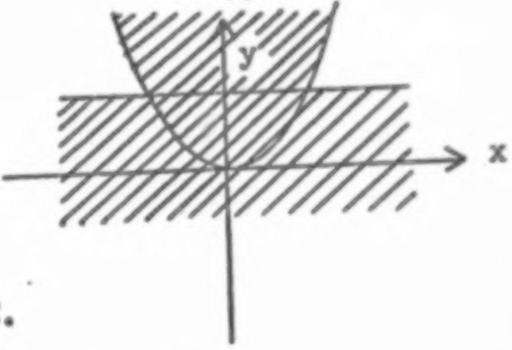

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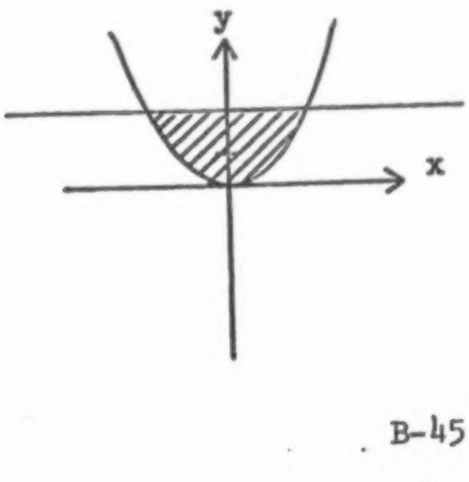
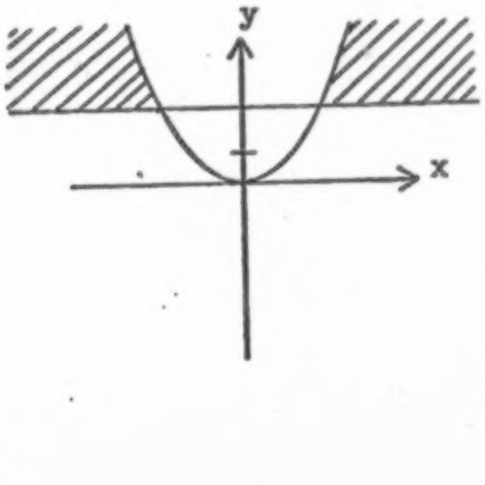


E.



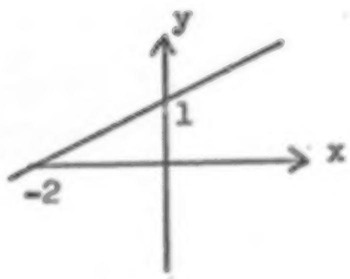
C

15. We wish to find a 2-digit number. The product of the two digits is 20 and their sum is 9. If the numbers were doubled, the resulting number would be a 3-digit number. What is this 2-digit number? (Hint: Let x designate one digit and y the other; then set up simultaneous equations.)

A. 54 B. 27 C. 45 D. 72 E. 36

16. Which one of the following is a graph of the equation $y = x + 1$? (Hint: try a few values of x, like $x = 0$ or $x = 1$ or $x = -1$.)

A.



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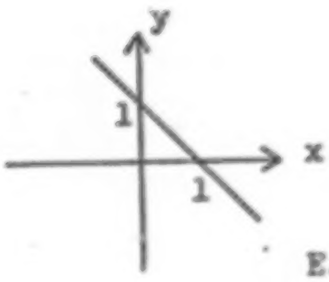

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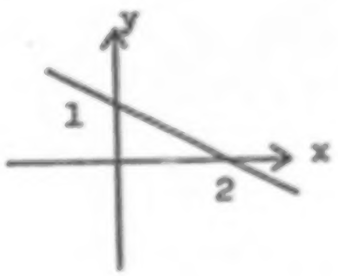

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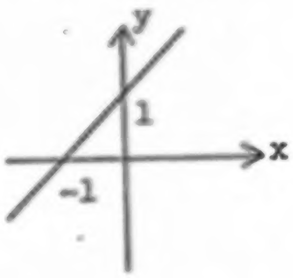

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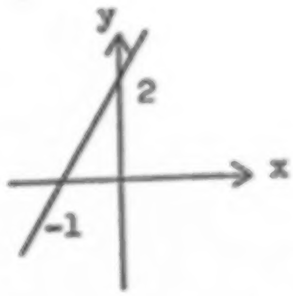
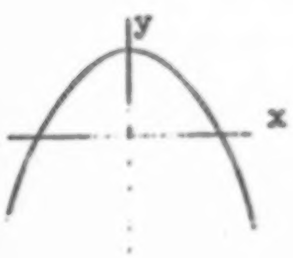
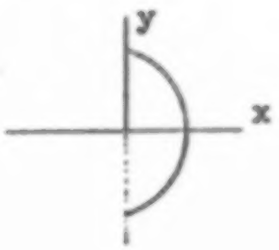


17. Definition: y is a function of x - in symbols, $y = f(x)$ - if there corresponds to each possible value of x only one value of y. Which one of the following cannot be a graph of $y = f(x)$?

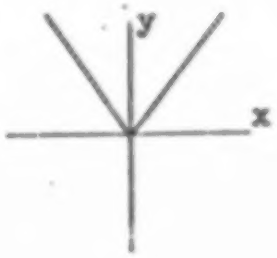
A.



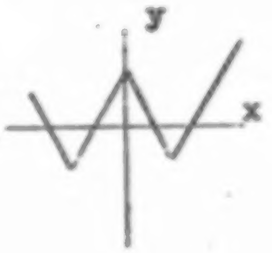
B.



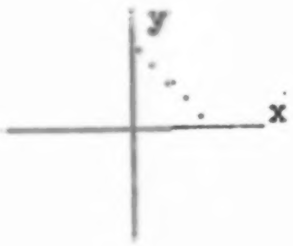
C.



D.



E.



18. Consider the table at the right defining the operation Δ. For example, a Δ b = c. To make this operation commutative, what letter or letters should be in the missing cell?

Δ	a	b
a	a	c
b		b

A. a B. b C. c D. a + b E. ab

19. To which one of the following expressions is $\frac{\sqrt{a^3b + ab^3}}{ab}$ equivalent?

A. $\frac{a + b}{ab}$ B. $\sqrt{a^2 + b^2}$ C. $\sqrt{\frac{a}{b} + \frac{b}{a}}$ D. $\sqrt{a + b}$ E. $a + b$

20. Working steadily and alone, John takes x days and Carl takes y days to build a fence. How many days would it take them if they each started at a different end and worked toward each other? (Hint: First find what fraction of the fence they could build together in one day.)

A. $\frac{x + y}{2}$ B. $\frac{1}{x + y}$ C. $\frac{1}{x} + \frac{1}{y}$ D. $\frac{1}{\frac{1}{x} + \frac{1}{y}}$ E. $x + y$

MINNESOTA NATIONAL LABORATORY

MATHEMATICS TEST

FORM D

(Grade Nine, 1965 Edition)

Read these instructions carefully.

Print your full name (last name first), grade (7, 8, 9, 10, 11, or 12), and sex (M or F) in the spaces provided on your IBM answer card.

Look at your IBM answer card. Note that there are two columns for your answers. One column is labeled 1st choice, and the other column is labeled 2nd choice. When you work a problem, you may obtain two answers, one which you think is most likely correct and another which you think is next most likely correct. If so, you are to mark the first answer in the column labeled 1st choice and the second answer in the column labeled 2nd choice. Even if you do not have a second choice, guess one and mark your guess in the column labeled 2nd choice. If you do not have a first or a second choice, guess them both and mark one guess in the column labeled 1st choice and the other guess in the column labeled 2nd choice. In other words, make sure that you mark two answers in the appropriate columns to every question. Using the electrographic pencil given you, make your marks heavily and so that they fill the entire spaces over and around the letters corresponding to your answers.

Do not write in the test booklet. Use scratch paper to do your figuring. You will have the remainder of the class hour to complete the test. If you have any questions, raise your hand; your teacher will come to you as soon as possible. Now begin.

1. If x is an even number, what is true of the expression $ax^2 + bx$?

   A. It is even if either a or b is even, but not both.

   B. It is odd if either a or b is odd, but not both.

   C. It is always odd.

   D. It is always even.

   E. It is odd if either a or b is even, but not both.

2. Which one of the following mathematical sentences is true if A, B, and C are not equal to zero?

   A. $\frac{A + B}{C} = \frac{A}{C} + \frac{B}{C}$ B. $\frac{1}{A} + \frac{1}{B} = \frac{1}{A + B}$ C. $\sqrt{A^2 + B^2} = A + B$

   D. $A(B + C)D = AB + CD$ E. $A(BC) = (AB)(AC)$

3. Definition: The set of numbers {a, b, c} is closed with respect to the arithmetic operation Δ if a Δ b, a Δ c, and b Δ c are equal to either a, b, or c.

   Which one of the following sets is closed?

   A. The set {0,1,2,3,...} with respect to subtraction.

   B. The set of perfect squares with respect to addition.

   C. The set {0} with respect to addition.

   D. The set of odd integers with respect to subtraction.

   E. The set of positive even integers with respect to division.

4. Look at the three equations below:

   1. $(x + 3)^2 = x^2 + 6x + 9$
   2. $x^2 - 3x = -2$
   3. $3 + x^2 = x^2 - 2$

Which one of the following statements about these equations is true?

A. 1 is true for some values of x, 2 is true for all values of x, and 3 is true for no values of x.

B. 1 is true for all values of x, 2 is true for some values of x, and 3 is true for no values of x.

C. Each equation is true for all values of x.

D. Each equation has exactly two solutions.

E. 1 and 3 are true for some values of x, and 2 is true for no values of x.

5. One fifth of the sum of 12 and a certain number equals one fourth of the sum of 6 and twice the number. The solution of which one of the following equations will yield the number?

A. $\frac{12}{5} + n = \frac{6}{4} + 2n$

B. $\frac{12 + n}{5} = \frac{6 + 2n}{4}$

C. $\frac{12 + n}{5} = \frac{3 + n}{4}$

D. $\frac{n}{5} + 12 = \frac{2n}{4} + 6$

E. $5(12 + n) = 4(6 + 2n)$

6. The formula $F = \frac{9}{5}C + 32$ may be used to convert a temperature reading on a Centigrade thermometer (C) to a temperature reading on a Fahrenheit thermometer (F). What is the temperature reading on a Centigrade thermometer when the reading on a Fahrenheit thermometer is -13°?

A. $-85^\circ C$ B. $-1^\circ C$ C. $9^\circ C$ D. $-8.6^\circ C$ E. $-25^\circ C$

7. Which one of the statements below is not always correct?

A. If $a = b$, then $ac = bc$.

B. If $a = b$, then $a + c = b + c$.

C. If $ac = bc$, then $a = b$.

D. If $ab = 0$, then either $a = 0$ or $b = 0$.

E. If $a = b$, then $a^2 = b^2$.

8. Which one of the following is always true if a, b, and c are positive integers (1,2,3,...)?

A. If $a > b$, then $\frac{c}{a} < \frac{c}{b}$

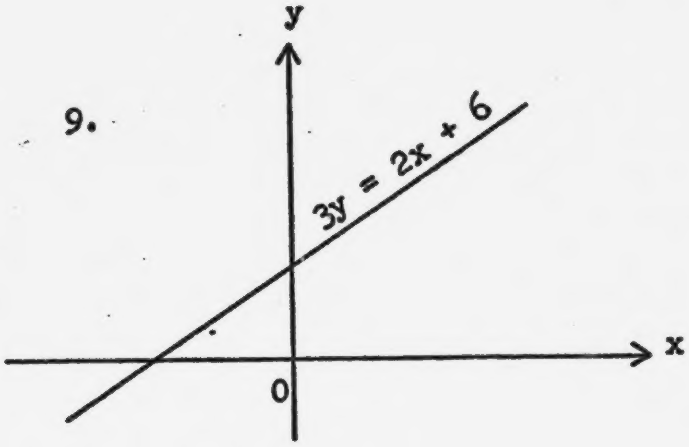
B. If $a > b > c$, then $\frac{1}{a} + \frac{1}{b} > \frac{1}{b} + \frac{1}{c}$

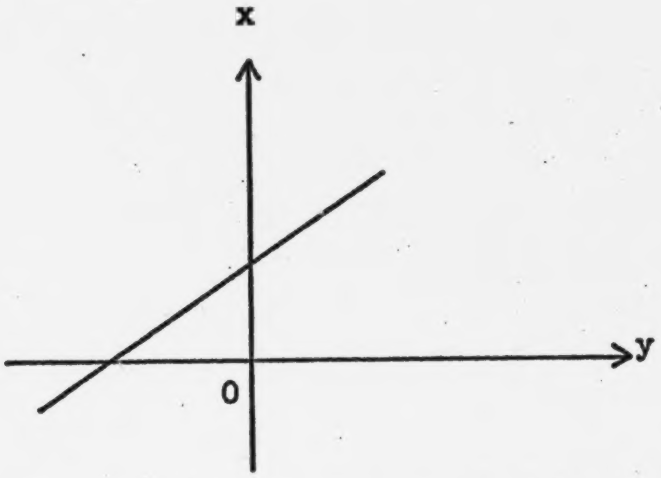
C. If $a > b > c$, then $\frac{1}{(a + c)} < \frac{1}{(a + b)}$

D. If $a > b > c$, then $\frac{b}{c} > \frac{a}{c}$

E. If $a > b$, then $\frac{1}{a} > \frac{1}{b}$

9.





The diagram at the left above is the graph of the line $3y = 2x + 6$. In the diagram at the right above the x and y axes have been reversed, but the line still remains in the same position. What is the equation of the line with reference to the axes in their new positions?

A. $3y = 2x + 6$

B. $2y = 3x + 6$

C. $3x = -2y - 6$

D. $3x = -2y + 6$

E. $3x = 2y + 6$

10. If
i) $f(x,y) = 1$ when $x > y$,
ii) $f(x,y) = 0$ when $x = y$, and
iii) $f(x,y) = -1$ when $x < y$,

what is the value of $f(3,0) + f(3,3) + f(0,3)$?

A. 12 B. 9 C. 0 D. 3 E. 6

11. For what value or values of x is the equation $x(x - 4) = 3(x - 4)$ true?

A. 3 B. 0 or 4 C. 3 or 4 D. 0 E. 0 or 3

12. The base of a triangle has the same length as the side of a square. A second side of the triangle is twice as long as the base, and the third side of the triangle is 12 inches shorter than three times the base. If the perimeter of the triangle equals that of the square, how many inches long is the base of the triangle?

A. 6 B. 12 C. 16 D. 24 E. 36

13. What value or values of x satisfy the inequality $\frac{5}{6} - \frac{1}{2} - 8x > \frac{1}{3}$?

A. $x = -\frac{1}{8}$ B. $x > \frac{1}{8}$ C. $x < 0$ D. $x > 0$ E. $x < -\frac{1}{8}$

14. Addition (+) and multiplication (·) are defined by the following tables:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

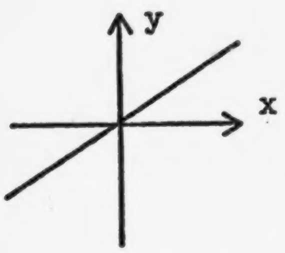
·	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

A linear equation $ax + b = c$ can be solved by using these tables. For example, if $x + 2 = 1$, then $x = 2$. Using these tables, what is the value of x in the equation $2x + 1 = 2$?

A. 0 B. 3 C. 1 D. 0 or 1 E. 2

15. Which one of the graphs below could be the graph of $y = \left|\frac{x}{4}\right|$?

A.



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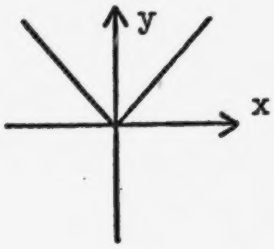

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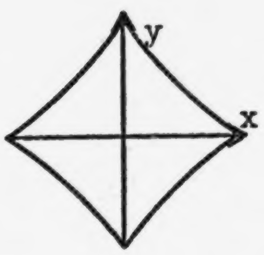

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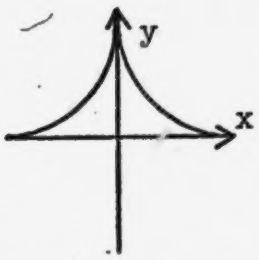
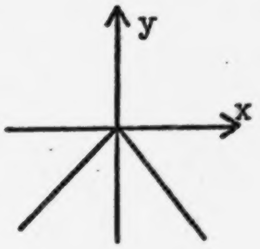


E.



16. If $(x - 1)(x - k) = x^2 + x + k$, what is the value of k?

A. 1 B. 2 C. 3 D. -2 E. -1

17. What values of x satisfy the inequality

$$(x + 2)(x - 3) < 0?$$

A. $x > -2$ B. $x \geq 3$ or $x \leq -2$ C. $x < -2$

D. $-2 < x < 3$ E. $x < 3$

18. What value or values of x satisfy both of the following inequalities?

$$x^2 < 9 \quad \text{and} \quad x + 3 \leq 1$$

A. $x = -2$ B. $x < 3$ C. $-3 < x \leq -2$ D. $-2 \leq x < 3$ E. $x < -2$

19. If $x \neq 0$, which one of the following is equivalent to $\dfrac{\sqrt{x} + \sqrt{4x}}{\sqrt{x}}$?

A. 3 B. $\sqrt{5}$ C. $x + 2$ D. $\sqrt{2x}$ E. 5

20. What is the sum of 7^{20} and 7^{21}?

A. 7^{41} B. 14^{41} C. 49^{420} D. $8(7^{20})$ E. $2(7^{41})$

MINNESOTA NATIONAL LABORATORY

MATHEMATICS TEST

FORM G

(Grade Eleven, 1964 Edition)

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1. If $|x - 1| = 2$, what number (or numbers) does x represent?

   A. 3 only B. -1 only C. -3 only D. -1 or 3 E. 2 only

2. What type of function is $f(x)$ if the sum of the coordinates is 5 for each and every point that lies on the graph of $f(x)$?

   A. A constant function.

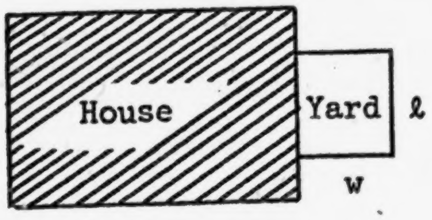
   B. A logarithmic function.

   C. A quadratic function.

   D. A cubic function.

   E. A linear function.

3. Suppose we have 60 feet of fencing available to enclose a rectangular yard that will have a house serving as one side of the yard. We notice (see table below) that the area enclosed will depend upon how this 60 feet of fencing is used.



Length	Width	Area
50 ft.	5 ft.	250 sq. ft.
40 ft.	10 ft.	400 sq. ft.
30 ft.	15 ft.	450 sq. ft.

   Which one of the following conclusions is correct?

   A. The maximum area will be obtained if the yard is square.

   B. The area of a rectangle is greater than the area of a square.

   C. The area of the yard continues to increase as its width increases.

   D. The maximum area will be obtained if the length is twice the width of the yard.

   E. The area can be made as large as we choose.

4. If $\log_y ab = 2$, what is the value of y?

   A. $\frac{ab}{2}$ B. 10 C. ab^2 D. 2ab E. $\sqrt{ab}$

5. What values of t make the expression $(t^2 - 1)^2x^2 + (t - 1)x + t + 1$ a quadratic function?

A. All real values of t.

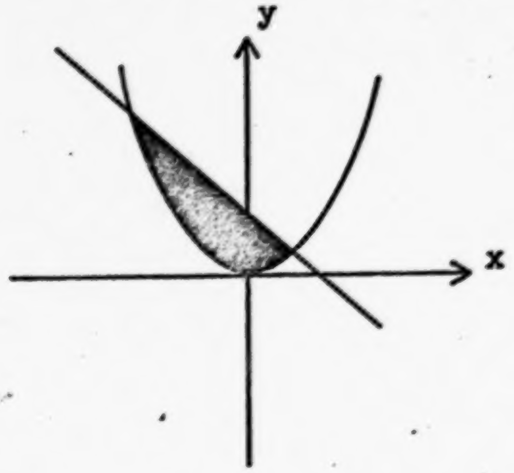
B. All real values of t except $t = 1$.

C. All real values of t except $t = -1$.

D. All real values of t except $t = 1$ and $t = -1$.

E. All real values of t except $-1 < t < 1$.

6. Which one of the following pairs of inequalities has a graphical solution as indicated by the shaded region of the graph at the right?



A. $x^2 \geq y$ and $y \leq x - 3$

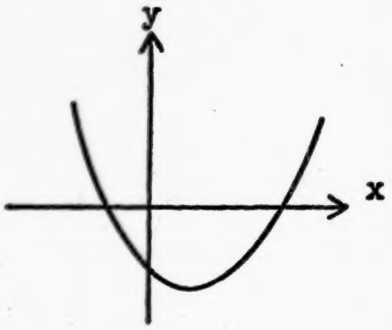
B. $x^2 \geq y$ and $y \geq 3$

C. $y \geq x^2$ and $y \leq 2x - 6$

D. $y \geq x^2$ and $y \leq 3 - x$

E. $x^2 \geq y$ and $y \leq 6 - 2x$

7. Consider the graph of the equation $y = x^2 - 2x - 3$ shown in the drawing. Which values of x satisfy the inequality $x^2 - 2x - 3 \geq 0$?



A. $x \geq 3$

B. $-1 \leq x \leq 3$

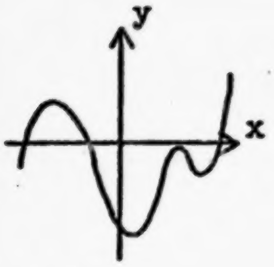
C. $x \geq -1$ or $x \geq 3$

D. $x \geq 3$ or $x \leq -1$

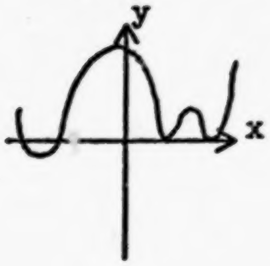
E. $x \leq -1$ or $x \leq 3$

8. Which one of the following is the most accurate representation of the graph of $f(x) = (x + 3)(x - 3)(x - 4)(x + 2)$?

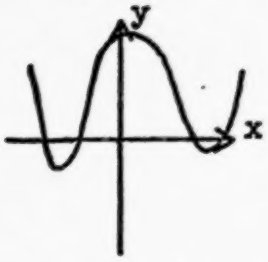
A.



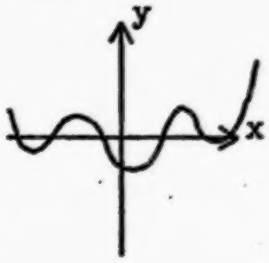
B.



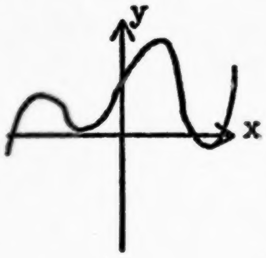
C.



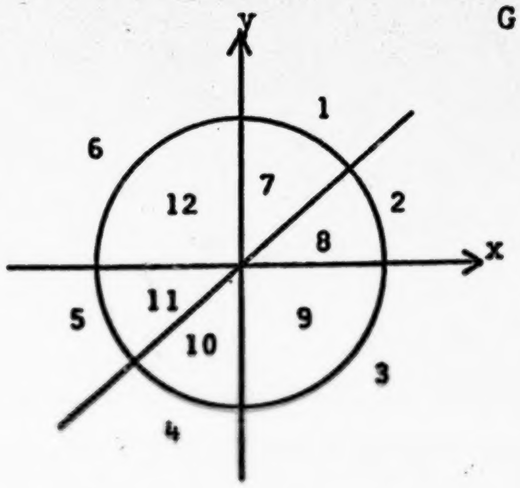
D.



E.



9. Note that the axes and graphs of the equations $x^2 + y^2 = 1$ and $x = y$ split the plane into twelve distinct regions, six of which are in the interior and six in the exterior of the circle. Which of these regions satisfy the inequalities



$$y > 0, \quad x^2 + y^2 > 1, \quad \text{and} \quad y > x \;?$$

A. 1, 2, 6 B. 1 only C. 1, 6 D. 7, 12 E. 2 only

10. If $f(x) = x + 7$, and $g(x)$ is the inverse of $f(x)$, what is $g(3)$?

A. $\frac{1}{10}$ B. -10 C. -4 D. $-\frac{1}{10}$ E. 4

11. What are the factors of $(a + 3)^2 - (b - 4)^2$?

A. $a + 3$ and $b - 4$
B. $a + b - 1$ and $a + b - 1$
C. $a + b + 1$ and $a - b - 7$
D. $a + b - 1$ and $a - b - 1$
E. $a + b - 1$ and $a - b + 7$

12. Which one of the following is equivalent to $\frac{\log (ab)}{\log a + \log b}$?

A. 1 B. 2 C. $\frac{ab}{a + b}$ D. $\log (ab) - \log a - \log b$ E. $\frac{\log (ab)}{ab}$

13. What is the value of $(5)(16^{\frac{1}{6}})(2^{\frac{1}{3}})$? (Hint: Start by substituting 2^4 for 16.)

A. $5\sqrt{32}$ B. $\sqrt{160}$ C. $5\sqrt[18]{32}$ D. 10 E. 20

14. If $x^2 > y^2$ and $y > 0$, which one of the following inequalities expresses a relationship between x and y that is always true?

A. $x > y$ B. $x > \pm y$ C. $x > \frac{y^2}{x}$ D. $x > y$ or $x < -y$

E. $x < y$ or $x > -y$

15. Consider the operation Δ. If $(a,b) \, \Delta \, (c,d) = (a - c, b + d)$ and $(3,2) \, \Delta \, (0,0) = (x,y) \, \Delta \, (3,2)$, to what is (x,y) equal?

A. $(0,0)$ B. $(-3,-2)$ C. $(3,2)$ D. $(6,0)$ E. $(2,3)$

16. How many terms are there in the product $(a + b)^7 \cdot (a + b)^8$?

A. 15 B. 16 C. 17 D. 57 E. 9

17. What must be true of k if the graph of the function $3x^2 + 2x + k$ intersects the x-axis? (Hint: Find the value of k for which the graph of the function intersects the x axis at one point; then draw the graph.)

A. $k = 0$ B. $k \geq 0$ C. $k \leq \frac{1}{3}$ D. $k \geq 1$ E. $k = -\frac{1}{3}$

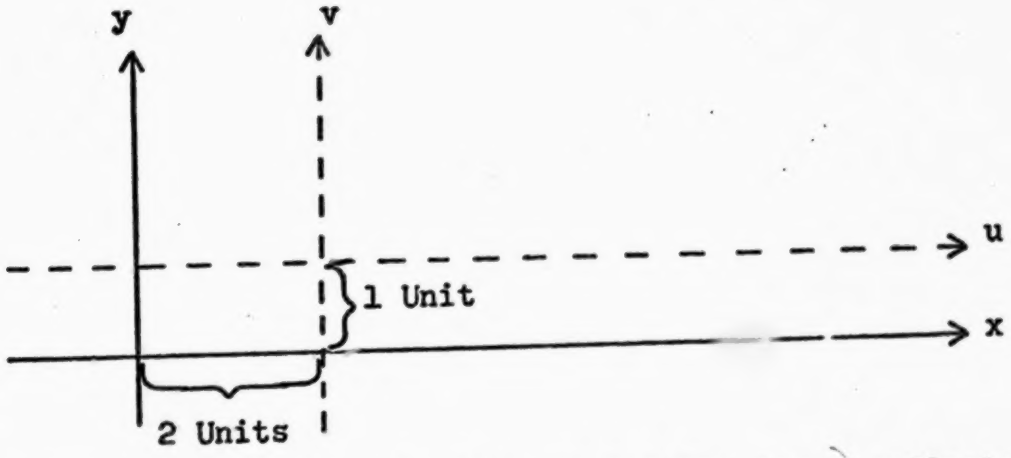
18. To which one of the following expressions is the product $(x^{-1} + y^{-1})(x + y)^{-1}$ equivalent?

A. $\frac{1}{xy}$ B. $\frac{1}{(x + y)^2}$ C. 1 D. $\frac{1}{x + y}$ E. $\frac{1}{x^2} + \frac{1}{y^2}$

19. The equation $y = x^2 - 4x + 3$ defines a function. If x may be any real number, what values may y be?

A. $y \geq 0$ B. $y \leq 0$ C. $y \geq -1$ D. $y \geq 3$ E. $y \geq -3$

20.



The equation of a line in the xy-rectangular coordinate system is $y = 2x + 4$. What is the equation of this line with reference to a uv-rectangular coordinate system, if the u-axis is parallel to the x-axis and 1 unit above it and if the v-axis is parallel to the y-axis and 2 units to the right of it? (Hint: Start by substituting $v + 1$ for y in the equation of the line.)

A. $v = 2u + 4$ B. $v = 2u + 5$ C. $v = 2u + 6$ D. $v = 2u + 7$

E. $v = 2u + 8$

Appendix C: Contingencies in content between Minnesota National Laboratory Course in Algebra for Teachers and:

1) Dolciani, M. P., Berman, S. L., & Freilich, J. Modern Algebra, Book One. Boston: Houghton Mifflin, 1962.

2) Dolciani, M. P., Berman, S. L., & Wooton, W. Modern Algebra and Trigonometry, Book Two. Boston: Houghton Mifflin, 1963.

MNL Algebra Course for Teachers

Topics (See key on Page C-12)

Dolciani et al. Book 1
Topics (See Key on Page C-14)

Unit		1								2						3				4			5	6	7	
Chapter		1	2	3	4	5	6	7	8	1	2	3	4	5	6	1	2	3	4	1	2	3	1	1	1	2
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Topics (See Key on Page C-12)

Dolciani et al. Book 1
Topics (See Key on Page C-14)

Chapter	Unit	8							9						10					11				12		13						14			
		1	2	3	4	5	6	7	1	2	3	4	5	6	1	2	3	4	5	1	2	3	4	1	2	1	2	3	4	5	6	1	2	3	4
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MNL Algebra Course for Teachers

Topics (See Key on Page C-12)

Dolciani et al. Book 1 Topics (See Key on Page C-14)	Unit	1								2						3				4			5	6	7	
Chapter		1	2	3	4	5	6	7	8	1	2	3	4	5	6	1	2	3	4	1	2	3	1	1	1	2
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MNL Algebra Course for Teachers

Topics (See Key on Page C-12)

Dolciani et al. Book 1 Topics (See Key on Page C-14)

	Unit	8							9						10					11				12		13						14			
Chapter		1	2	3	4	5	6	7	1	2	3	4	5	6	1	2	3	4	5	1	2	3	4	1	2	1	2	3	4	5	6	1	2	3	4
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MNL Algebra Course for Teachers (continued)

Topics (See Key on Page C-12)

Dolciani et al. Book 1 Topics (See Key on Page C-14)

Chapter	Unit	1								2						3				4			5	6	7	
		1	2	3	4	5	6	7	8	1	2	3	4	5	6	1	2	3	4	1	2	3	1	1	1	2
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Topics (See Key on Page C-12)

Dolciani et al. Book 1
Topics (See Key on Page C-14)

Unit		8							9						10					11				12		13						14			
Chapter		1	2	3	4	5	6	7	1	2	3	4	5	6	1	2	3	4	5	1	2	3	4	1	2	1	2	3	4	5	6	1	2	3	4
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Topics (See Key on Page C-12)

Dolciani et al. Book 2
Topics (See Key on Page C-16)

Chapter	Unit	1								2						3				4			5	6	7	
		1	2	3	4	5	6	7	8	1	2	3	4	5	6	1	2	3	4	1	2	3	1	1	1	2
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MNL Algebra Course for Teachers Topics (See Key on Page C-12)

Dolciani et al. Book 2
Topics (See Key on Page C-16)

Chapter	Unit	8							9						10					11				12		13						14			
		1	2	3	4	5	6	7	1	2	3	4	5	6	1	2	3	4	5	1	2	3	4	1	2	1	2	3	4	5	6	1	2	3	4
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MNL Algebra Course for Teachers

Topics (See Key on Page C-12)

Dolciani et al. Book 2 Topics (See Key on Page C-16)

	Unit	1								2						3				4			5	6	7	
Chapter		1	2	3	4	5	6	7	8	1	2	3	4	5	6	1	2	3	4	1	2	3	1	1	1	2
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Topics (See Key on Page C-12)

Dolciani et al. Book 2 Topics (See Key on Page C-16)

Unit		8							9						10					11				12		13						14			
Chapter		1	2	3	4	5	6	7	1	2	3	4	5	6	1	2	3	4	5	1	2	3	4	1	2	1	2	3	4	5	6	1	2	3	4
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Key to Topics from MNL Algebra Course for Teachers

Frequency of Contingencies with Topics from Dolciani et al.

	Book 1			Book 2		
	A	B	C	A	B	C
Unit 1: Sets, Relations, and Functions						
Sets		3	2	2		7
Ordered Pairs and Cartesian Products			3			7
Relations		3		1	1	7
Functions		2	1	1	1	5
Functional Notation				1	1	2
Inverse of a Function						1
Composition of Functions					1	
Implicitly Defined Functions						
Unit 2: Algebra of Real Numbers						
Operations			4		1	
Properties of Real Number Operations		6	3		1	4
Field Theorems for the Real Number System		5	6		1	5
Modular Arithmetics			1			1
Definition of a Group						
Subtraction and Division		4	3		1	6
Unit 3: Algebraic Systems						
Theorems on Additive and Multiplicative Inverses		2	4		3	5
Matrices				2		2
Theorems on Fractions		2	2		1	2
Isomorphism of Systems						
Unit 4: Order in the Real Number System						
Order Postulates		3	1		1	1
Order Properties		1	1		1	3
Order Theorems					1	1
Unit 5: Equations and Inequalities						
Solution of Equations and Inequalities		8	5		6	1
Unit 6: Absolute Value						
Absolute Value		1			1	
Unit 7: Completeness of the Real Number System						
Completeness of the Real Number System			1		1	1
Decimal Expansions						1

	Frequency of Contingencies with Topics from Dolciani et al.					
	Book 1			Book 2		
	A	B	C	A	B	C
Unit 8: Natural Numbers						
Definition of the Set of Natural Numbers						1
Well-Ordering and Mathematical Induction					1	
Alternate Definition of N						1
Prime Numbers			1			2
Prime Factorization			1		1	1
Countability					1	
The Archimedean Property						1
Unit 9: Integers						
Definition of the Set of Integers		1	4			1
Division with Remainder		1				1
Decimal Representation and Use of Other Bases						
Euclid's Algorithm					1	
Prime Integers			1			
Countability						
Unit 10: Rational Numbers						
Definition of the Set Q		1	1			2
Operations and Their Properties		1	1			2
Properties of the System of Rational Numbers						3
Completeness and the Rational Numbers		1	2			2
Comparison of Systems of Numbers		2				
Unit 11: Complex Numbers						
Complex Numbers: Definition and Basic Operations	1			1		3
Isomorphism of a Subsystem of C with the System of Real Numbers						
Conjugate of a Complex Number						
Graphing and Absolute Value						2
Unit 12: Algebra of Real Functions						
Algebra of Real Functions			1			
Polynomial Functions			3			
Unit 13: Polynomials						
Polynomials		3	1		1	1
Algebra of Polynomials		3			2	2
Division Theorem for Polynomials, Factoring		1			2	1
Remainder and Factor Theorems		1			1	
Rational and Complex Polynomials			3		2	1
Polynomials over other Fields			3			1
Unit 14: Equivalence Relations and Groups						
Properties of Relations						
Equivalence Relations and Equivalence Classes						
Groups			4			
Other Properties of a Group			4		1	2

Key to Topics from Dolciani et al. Book 1

	Frequencies of Contingencies with Topics from MNL Algebra Course		
	A	B	C
Chapter 1: Symbols and Sets			
Numbers - Order Relations			1
Equality		1	
Inequality		1	
Grouping Numbers in Sets		1	
Punctuation Marks in Algebra			1
Extra: Arithmetic of Sets: Intersection		1	
Chapter 2: Variables and Open Sentences			
Evaluating Algebraic Expressions			1
Factors, Coefficients, and Exponents			2
Solving Open Sentences		1	
Thinking with Variables			
Solving Problems - Open Sentences			
Extra: Sets - Union of		1	1
Chapter 3: Axioms, Equations, and Problem Solving			
Axioms of Equality		1	
Addition Properties of Numbers		4	2
Multiplication, Division Properties of Numbers		4	2
Solving 1st Degree Equations		1	3
Extra: Algebra of Logic and Sets			3
Chapter 4: The Negative Numbers			
Directed Numbers		1	
Comparing Numbers (<,>)		1	
Addition on Number Line			2
Opposite of a Number (Inverse)		1	4
Absolute Value		1	
Subtracting Directed Numbers		1	3
Multiplying Directed Numbers			4
Dividing Directed Numbers		2	3
Averages			
Extra: Clock Arithmetic			1
Chapter 5: Equations, Inequalities, and Problem Solving			
Transforming Equations		1	
Properties of Inequality		2	
Pairs of Inequalities		1	
Analysis of Problems			
Number System Structure		1	
Extra: Systems of Numeration			
Chapter 6: Working with Polynomials			
Adding and Subtracting Polynomials		2	3
Multiplication of Polynomials		2	3
Division of Polynomials		3	3
Extra: Negative Exponents			

	Frequencies of Contingencies with Topics from MNL Algebra Course		
	A	B	C
Chapter 7: Special Products and Factoring			
Distributive Property - Factoring		2	
Quadratic Trinomials		1	
Extension of Factoring		2	
Solving Equations by Factoring		1	1
Extra: Scientific Notation			
Chapter 8: Working with Fractions			
Fractions and Ratios (Algebraic)			4
Per Cent			
Multiplying and Dividing Fractions		1	
Adding and Subtracting Fractions		1	
Complex Fractions			1
Fractions in Open Sentences			
Extra: Divisibility of Integers			
Chapter 9: Graphs			
Ordered Pairs - Points in Plane		1	1
Linear Equations - Straight Lines		2	
Inequalities and Special Graphs			2
Extra: The Sieve of Eratosthenes			2
Chapter 10: Sentences in Two Variables			
Solving Systems - Linear			1
Pairs of Linear Inequalities			1
Additional Problems			
Extra: Diophantine Equations			
Chapter 11: The Real Numbers			
System of Rational Numbers		2	1
Irrational Numbers		1	
Radical Expressions			
Real Number System		1	6
Radical Equations			1
Extra: Algebraic Fallacies			
Chapter 12: Functions and Variation			
Relations and Functions		2	1
Variation - Direct and Inverse			2
Extra: Linear Programming			
Chapter 13: Quadratic Equations and Inequalities			
General Methods - Solving Quadratic Equations		1	
Solution of Quadratic Inequalities		1	
Extra: Factor Theorem		1	
Chapter 14: Geometry and Trigonometry			
Geometry			
Trigonometry			
Vectors			
Extra: Complex Numbers	1		

Key to Topics from Dolciani et al. Book 2

	Frequencies of Contingencies with Topics from MNL Algebra Course		
	A	B	C
Chapter 1: Sets of Numbers; Axioms			
Sets	1		
Set Relations	1		6
Variables		1	3
Real Number Field Axioms		3	
Real Number Theorems		5	
Chapter 2: Open Sentences in One Variable			
Equivalent Expressions		1	
Equivalent Equations		1	4
Equivalent Inequalities		4	2
Combining Expressions		1	1
Solving Word Problems			
Extra: More about Proof			
Chapter 3: Systems of Linear Open Sentences			
Linear Equations and Their Graphs			3
Linear Equations - Properties of Lines			
Linear Inequalities		1	5
Extra: Linear Programming			
Chapter 4: Polynomials and Factoring			
Products of Polynomials		2	3
Factoring Polynomials		1	6
Greatest Common Factors and Least Common Multiples		1	2
Solving Equations by Factoring			1
Division of Polynomials		1	1
Extra: Euclid's Algorithm		1	
Chapter 5: Rational Numbers and Expressions			
Zero and Negative Exponents			1
Rational Algebraic Expressions		1	7
Rational Number Operations		1	6
Polynomials - Rational Coefficients			4
Fractional Equations			5
Extra: Congruences			1
Chapter 6: Relations and Functions			
Relations	1		1
Functions	2	1	2
Linear Functions		1	2
Quadratic Functions and Relations			3
Extra: Denumerability		1	
Chapter 7: Irrational Numbers and Quadratic Equations			
Real Roots of Real Numbers		1	4
Radicals - Operations			
Quadratic Formula			3
Quadratic Inequalities			1
Extra: Continued Fractions			

	Frequencies of Contingencies with Topics from MNL Algebra Course		
	A	B	C
Chapter 8: Quadratic Relations and Systems			
Distance in Plane			
Quadratic Relations - Conic Section		1	2
Solving Quadratic Systems		1	
Extra: Conic Sections			
Chapter 9: Exponential Functions and Logarithms			
Rational and Real Exponents			
Exponential Functions			
Logarithmic Functions			
Calculation Using Logs			
Chapter 10: Trigonometric Functions and Complex Numbers			
Trigonometric Functions			2
Applying Trigonometric Functions			
Adding and Resolving Vectors			
Complex Numbers	1		
Complex Numbers and Quadratic Equations			2
Chapter 11: Trigonometric Identities and Formulas			
Trigonometric Identities			
Extra: Trigonometric Form of Complex Numbers			
Chapter 12: The Circular Functions and Their Inverses			
Circular Functions			2
Inverse Trigonometric Functions			1
Extra: Irrational Trigonometric Values and Logarithms			
Chapter 13: Progressions and Binomial Expansions			
Arithmetic Progressions			
Geometric Progressions			
Geometric Series			1
Binomial Expansion			1
Extra: Mathematical Induction		1	
Chapter 14: Polynomial Functions			
Polynomial Functions - Complex Numbers		1	1
Remainder and Factor Theorems		2	
Fundamental Theorem of Algebra		1	
Extra: Analytic Proofs			
Chapter 15: Matrices and Determinants			
Matrix Operations	1		
Determinant of a Matrix			
Inverse of a Matrix (2 x 2)	1		
Solving of Linear Systems by Determinants			1
Cramer's Rule			1

	Frequencies of Contingencies with Topics from MNL Algebra Course		
	A	B	C
Chapter 16: Permutation, Combinations, and Probability			
Permutations			1
Combinations			
Probability			

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TITLE
A Study of the Relative Effectiveness of High School Algebra Teachers in Relation to Measures of Their Proficiency in Foundations of Modern Algebra. Final Report.

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IDENTIFIERS Ball State Teachers College Experimental Mathematics Program, University of Illinois Committee on School Mathematics, School Mathematics Study Group.

ABSTRACT

Ninth and eleventh grade teachers were followed as they began to teach a new mathematics curriculum (SMSG, Illinois, or Ball State). Their classes were tested for several consecutive years with a variety of achievement tests. Some teachers were given a programmed correspondence course in foundations of modern algebra.

Rank orders of the teachers by gains of their classes were significantly correlated across successive years: some were consistently more effective than others. When the same teachers taught both modern and conventional classes, their rank order by gains in modern classes were significantly correlated with their rank order by gains in conventional classes. Teacher effects tended to dominate curriculum effects. Mild but consistent relations were found between gains of classes and the relative speed and thoroughness with which teachers themselves learned new mathematics content through the programmed correspondence course.